CUPM Recommendations: The First Forty Years

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Context for Action

Ferment in the Monthly About Collegiate Mathematics

During the five year period 1953-57 at the beginning of the Eisenhower administration and after the end of the Korean War, the *American Mathematical Monthly* manifested mathematicians' dissatisfaction with collegiate mathematics in these early pre-Sputnik years of the "Cold War." A string of reports, reviews, and arguments focused on the need for, in the words of William Duren, a "much-talked about revolution in the mathematics program." The papers linked below, listed chronologically, represent the ferment of this period:

- The Mathematical Training of Engineers. Ralph Hull. *Amer. Mathematical Monthly*, 60:2 (Feb. 1953) 106-108. Report of presentations and discussion at a 1952 MAA symposium, the second in a series on the changing content of the undergraduate mathematics major.
- Some Observations on Undergraduate Mathematics in American Colleges and Universities. E.A. Cameron. *Amer. Mathematical Monthly*, 60:3 (Mar. 1953) 151-155. Report of a year-long Ford Foundation-sponsored project in which Cameron visited and described mathematics departments in 33 colleges and universities of the "highest scholastic standing" across the United States.
- Teacher Education in Algebra. C.C. MacDuffee. Amer. Mathematical Monthly, 60:6 (Jun. 1953) 367-375. Lectures from a 1952 Symposium on Teacher Education at the University of Wisconsin

responding to entering students' "very spotty preparation in algebra ... even among those who enter

the engineering college."

- Mathematics in School and College. Amer. Mathematical Monthly, 60:6 (Jun. 1953) 380-383. Extract from General Education in School and College, Alan R. Blackmer, editor, Harvard University Press, 1952. Based on consultation with faculty in elite secondary schools and colleges and "framed with special reference to the better-than-average student," this study reports "a very high degree of consensus" that the standard high school mathematics curriculum (two years of algebra, one of plane geometry, and one of solid geometry and trigonometry) that has "the sanction of the ages" is ready for "drastic alteration and improvement." This study recommends reducing repetition in order to introduce calculus and statistics that will prove useful in college. "The case for statistics," it concludes, "is even more powerful than the case for calculus."
- Elementary and Secondary School Training in Mathematics. S.S. Cairns. Amer. Mathematical Monthly, 60:8 (Oct. 1953) 523-527. Testimony to a 1952 Illinois "School Problems Commission" concerning "the national emergency and associated critical shortage of scientifically trained manpower."
- The NRC-AMS Conference on Training in Applied Mathematics. F. Joachim Weyl. Bulletin Amer. Math. Society, 40:1 (Jan. 1954) 38-44. A summary report of a conference designed "to subject the possible patterns of training applied mathematicians to an intensive discussion ... by both producers and consumers of applied mathematicians."

• Mathematical Teaching in Universities. Andre Weil. *Amer. Mathematical Monthly*, 61:1 (Jan. 1954) 34-36. Nine pithy precepts (with corollaries) for effective development of mathematical expertise. • Of Course and Courses. Saunders Mac Lane. *Amer. Mathematical Monthly*, 61:3 (Mar. 1954) 151-154. In this retiring MAA Presidential Address, Mac Lane urges the design of "modern and coherent curricula" that "exhibit the unity of mathematics" devoid of "traditional impedimenta." • Freshman Mathematics as an Integral Part of Western Culture. Morris Kline. *Amer. Mathematical Monthly*, 61:5 (May 1954) 295-306. An argument for "appreciation rather than skill" as the objective of mathematics courses offered to students who do "not intend to use mathematics in some profession or career."

- Mathematics in the Secondary Schools for the Exceptional Student. H.W. Brinkman. Amer. Mathematical Monthly, 61:5 (May 1954) 319-323. Part of a multi-subject School and College Study of Admission with Advanced Standing undertaken with support from the Fund for the Advancement of Education. This study, a precursor to the Advanced Placement (AP) program, suggested a revision in the curriculum of grades 10-12 for accelerating students designed so that selective colleges would be willing to give credit for the 12th grade high school course. Their proposal would (a) expand the deductive part of 10th grade Euclidean geometry to apply to algebra and other areas of mathematics, (b) focus the 11th grade on analysis (algebra, analytic geometry, trigonometry), and (c) offer a "substantial introduction" to differential and integral calculus in the 12th grade.
- Special Topics in Applied Mathematics: Introduction and Critique. F. J. Weyl. Amer. Mathematical Monthly, 61:7 Part 2 (Sep. 1954) 1-4. Introduction to a Slaught Memorial Paper containing proceedings of a symposium on "Special Topics in Applied Mathematics," including electric circuit design, signal and noise problems, hydrodynamics, singularities on shocks, and vibrations of a crystal lattice.
- Mathematics for Social Scientists. R.R. Bush, W.G. Madow, Howard Raiffa, R. M. Thrall. Amer. Mathematical Monthly, 61:8 (Oct. 1954) 550-561. Report of a round–table discussion by the faculty of the 1953 Summer Institute of Mathematics for Social Scientists sponsored by the Social Science Research Council. The Institute introduced social scientists to sets, axiomatics, calculus, linear

algebra, probability, stochastic processes, and models (e.g., game theory, linear programming). In this paper four members of the Institute faculty discuss the rationale and goals of this course and propose it as a desirable alternative "for most students" to the then-standard calculus-only course sequence in the first two years of college mathematics.

- The First Conference on Training Personnel for the Computing Machine Field. Franz Hohn. *Amer. Mathematical Monthly*, 62:1 (Jan. 1955) 8-15. Report of a June 1954 conference enumerating personnel requirements of the newly emerging computer field together with desired features of educational programs that recruit and train students for this field.
- Joint Committee of the American Society for Engineering Education (ASEE) and the Mathematical Association of America (MAA) on Engineering Mathematics. R.S. Burington, Jr. Amer. Mathematical Monthly, 62:5 (May 1955) 385-392. Report of a liaison committee between MAA's new CUP and ASEE's Mathematics Division chaired by R.S. Burrington, Jr. Recommendations focus on what mathematics should be taught to students of engineering, who should teach it, and how it should be taught.
- Mathematical Consultants, Computational Mathematics and Mathematical Engineering. J.W. Tukey. *Amer. Mathematical Monthly*, 62:8 (Oct. 1955) 565-571. Observations about "attitudes" which mathematics departments might take towards the training of mathematics consultants for industry, the mathematics of computation, and the structure of educational programs in applied mathematics.
- The Teaching of Concrete Mathematics. J.W. Tukey. Amer. Mathematical Monthly, 65:1 (Jan. 1958)
 1-9. A companion to Tukey's 1955 essay, both being outgrowths of an October 1953 conference on training in applied mathematics (see F. J. Weyl's *Bulletin AMS* article cited above). In this commentary, Tukey reflects on a "syndrome never discussed in open meeting," namely, that while "applied mathematics' is more difficult than 'pure mathematics' in requiring more maturity and more years of study, ... the students who study in 'applied' fields do not compare in strength with those who go into 'pure' mathematics."

CUP and Universal Mathematics

The First Project in MAA's Response to This Ferment

Soon after the Committee on the Undergraduate Program (CUP) began its work, it compiled a litany of issues facing undergraduate mathematics, including

- "the rigid sequential organization of the traditional program with each course depending heavily on technical prerequisites,"
- "the growing tendency to repeat high school courses in college," and
- "[today's] emphasis upon cultural and liberal aspects in education."

Undergirding "this complex of inertial elements," according to the new CUP, was "the desertion of elementary teaching by the best mathematicians, old and young."

Arguing that in school and college mathematics new ideas have usually been "largely overpowered by the

self-propagation of the traditional but retrogressive stock," CUP's first major decision was "to seek one, universal freshman course for all reasonably qualified students." The Committee reasoned that

... only by means of such a universal course can the best principles of liberal education be served. Only in this way can we avoid the error of forcing the immature student, upon entering college, to make choices which will seriously restrict his freedom of development in later years.

This new course CUP called *Universal Mathematics*. The full rationale and a proposed outline for this course can be found in the Committee's first report to the Association by W.L. Duren, Jr., *et al.* [*Amer. Mathematical Monthly*, 62:7 (Sept. 1955) 511-520.]

Since the purpose of CUP was to promote rather than implement reform, its action on this matter was to encourage (rather than to create) this proposed new course. ("This Committee will not get into the textbook business and will not approve or disapprove any textbook.") Two summer writing groups associated with (but not appointed by) CUP took up the challenge of Universal Mathematics. One, meeting at the University of Kansas during the summer of 1954, focused on the first half of the CUP outline. A mimeographed preliminary edition—not a text, but "a book of experimental text materials"—entitled *Universal Mathematics, Part I: Functions and Limits* was published by the University of Kansas in September, 1954 (see the announcement by Carl B. Allendoerfer in the *Monthly*, 61:12 (Dec. 1954) p. 726.)

A second Writing Group met at Tulane University during the following summer. This team focused on sets and axioms—the second half of the CUP outline. A mimeographed preliminary edition, *Universal Mathematics, Part II: Elementary Mathematics of Sets*, Robert L. Davis, editor, was published by Tulane University in September, 1955 (see this announcement in the *Monthly*, 62:7 (Sep. 1955) p. 500.) Three years later, in 1958, MAA published a revised version of this volume under the title *Elementary Mathematics of Sets With Applications*.

The 1954 Summer Writing Group reports in the Preface to *Universal Mathematics* that in order to make the course "available to the maximum number of students," it was designed for "all first-year college and university students" with "at least two, and preferably two and one-half, units of high school mathematics, including intermediate algebra." This preparation the authors described as "normal" high school preparation. (For students of engineering, they suggested a supplementary "Technical Laboratory" to develop the problem–solving skills needed by such students.)

As the Preface notes, the book's "most striking feature" is its "dual presentation in two parallel lines of discourse." One offers "essentially all of its own" definitions and assumptions, but contains "only intuitive indications of proofs." The other, a formal presentation is "quite difficult" and is "not intended as a text to be followed continuously" but as a resource from which the instructor can "select those proofs which are appropriate for his students."

The 1955 Summer Writing Group at Tulane continued the dual presentation strategy from *Universal Mathematics, Part I.* Although *Part II* was originally conceived as a second semester course to follow *Part I*, the Tulane authors designed *Part II* to be independent of that course. Consequently, *Part II* "could be the first semester course in the first college year, without serious omissions."

In contrast to existing texts that treat set theoretic ideas "entirely in a logical and somewhat recondite manner," *Universal Mathematics* seeks to connect "the ideas of sets with a wide variety of subjects in science, engineering, behavioral studies, as well as pastimes and games." In so doing, it "lays a foundation for statistics

"without entering into statistics proper." In their Preface, the authors of *Part II* note that students' lack of experience with sets seriously limits their ability "to grasp the ideas of choice and chance, and of statistics." In turn, this "lack of training in these modes of thought is a handicap in many aspects of science, business and government."

Notwithstanding their sense of urgency, the Tulane authors note with caution that "The effort to present a variety of mathematics subjects in a framework of a theory of sets is frankly experimental. Its value remains to be tested."

A review of *Universal Mathematics, Part I* by Herbert (H.P.) Evans of the University of Wisconsin appeared in the *Monthly* in 1956 (pp. 196-199) followed by a report (pp. 199-202) by CUP chair William (W.L.) Duren, Jr., on a "mass trial" with 750 first year students at Tulane University. Evans commends the authors for making a "serious effort "to present real mathematics at an elementary level" and for treating a classical subject "from a fresh and modern viewpoint." He worries, however, that "even the intuitive treatment will be difficult for most students to follow and that the instructor will not always find it easy to present the material effectively."

Duren's report of the Tulane trial confirmed Evans' worries. "The book ... caused considerable difficulty to students and instructors. The main trouble is that students cannot read it." *Universal Mathematics* is "not adaptable to students whose high school background in mathematics is scant." Duren cites many factors that contribute to these difficulties, one of which is the "unusual method by which it was uncompromisingly written." Only after the formal account was complete was the intuitive version prepared, "written to follow [the formal account] closely in the same order." This approach—when "mathematical theory determines the formulations of concepts and their order of introduction—requires of instructors "much more imagination to find appropriate motivations and interpretations for young students."

(It is probably worth noting here that William Duren was chair of the Committee on the Undergraduate Program (CUP) that developed *Universal Mathematics*, was a member of the 1954 Summer Writing Group at the University of Kansas that developed *Part I*, and is the author of this report on the difficult experience of using this draft for first year students at Tulane.)

A few months after this review and report on Universal Mathematics appeared, the *Monthly* also published a contrasting strategy for first-year college mathematics in a report entitled College Mathematics for Non-Science Majors prepared by a special subcommittee of the California Committee for the Study of Education. The final section of this report offers comments about the CUP proposal for *Universal Mathematics*. While the California committee supports the CUP goal of a single introductory course for students interested in the "technical mathematics" of "natural science, social sciences or the arts," it sees these students as quite distinct from those for whom a course in mathematics for general education is intended:

We feel that it would be inadvisable to give the technically able students, with superior mathematical background of high school mathematics, the same course as average liberal arts students pursuing non-scientific courses, who have had little training in secondary mathematics.

To meet a perceived demand for early specialization, CUP suggested a curricular division in mathematics following *Universal Mathematics*: one course for physical science and engineering majors and a quite different one for those more interested in the biological and social sciences. In 1958 a Writing Group at Dartmouth College produced *Modern Mathematical Methods And Models*, a two-volume set of experimental text materials intended for the second of these CUP-proposed courses. The first volume, *Multicomponent Methods*, begins

with matrix methods which are then used to study functions of several variables. The second volume, *Mathematical Models*, introduces probability, order relations, Markov chains, and mathematical models.

Recognizing that many of the ideas in this course "would previously have been considered too difficult for a sophomore course," the authors nonetheless argue that students of biological and social science need to learn "appropriate mathematical tools early enough to use them," and therefore these mathematical ideas "must somehow be brought to a more elementary level." The first volume received a favorable brief review in the *Monthly* in March, 1959 (pp. 246-247).

Both *Universal Mathematics* and *Modern Mathematical Methods And Models* were developed by writing groups in support of CUP's call for innovation in undergraduate courses because for these two courses there were few, if any, existing texts. In contrast, there were many texts available that covered the topics in CUP's other sophomore course for students of physical science and engineering. So no special writing group was needed for this course, although CUP did put forward as a model of excellence George B. Seligman's notes on a course taught by Emil Artin to the top ten or fifteen percent of freshmen at Princeton University. According to the *Monthly* review, *A Freshman Honors Course in Calculus and Analytic Geometry Taught at Princeton University* (as these notes were titled) "should certainly be on the active bookshelf of every mathematician interested in teaching gifted students."

In summary, during its five years of work from 1953 through 1958, MAA's Committee on the Undergraduate Program (CUP), supported and distributed (without charge) five volumes intended to spark innovation in freshman and sophomore mathematics:

- Universal Mathematics, Part I: Functions and Limits. F.A.I. Bowers, Jr., R.N. Bradt, C.E. Capel, W.L. Duren, Jr., G.B. Price, W.R. Scott. University of Kansas, 1954.
- Universal Mathematics, Part II: Elementary Mathematics of Sets. R.N. Bradt, C.E. Capel, W.L. Duren, Jr., J.G. Kemeny, E.J. McShane, D.R. Morrison, G.B. Price, A.L. Putnam, W.R. Scott, & A.W. Tucker. Tulane University 1955. Revised as *Elementary Mathematics Of Sets With Applications*. R.N. Brandt & R.L. Davis. Mathematical Association of America, 1958.
- A Freshman Honors Course in Calculus and Analytic Geometry Taught at Princeton University. Emil Artin & George B. Seligman. Committee on the Undergraduate Program, Mathematical Association of America, 1957; University of Buffalo, 1958.
- Modern Mathematical Methods And Models, Volume I: Multicomponent Methods. E.J. Cogan, R.L. Davis, J.G. Kemeny, R.Z. Norman, J.A. Snell, & G.L. Thompson. Committee on the Undergraduate Program, Mathematical Association of America, 1958.
- Modern Mathematical Methods And Models, Volume II: Mathematical Models. Cogan, E.J., Kemeny, J.G., Norman, R.Z., Snell, J.L., & Thompson, G.L. Committee on the Undergraduate Program, Mathematical Association of America, 1958.

Since these volumes fulfilled the committee's original mandate, CUP also prepared several archival reports of its work:

• Report of the Committee on the Undergraduate Mathematical Program. W.L. Duren *et al. Amer. Mathematical Monthly*, 62:7 (Sep. 1955) 511-520. An interim report describing the work of the committee and highlighting experimental courses that address some of the issues on the committee's agenda.

- Collected Reports of the CUP. Committee on the Undergraduate Program, W.L. Duren, Jr., editor. University of Virginia, 1957.
- Transition Report and Outline of Recommended Courses. R.L. Davis. University of Virginia. 1958.

The Founding of CUPM 1958-1978: Part I

Supported by NSF, the New CUPM Proposed Numerous Curricular Changes

In 1958 the National Science Foundation awarded MAA a grant to convene a broadly representative group of mathematicians and users of mathematics "to study some of the problems created by the revolution in mathematics and to seek solutions for them." These problems included rapid increases in enrollments, unprecedented demand for mathematics teachers, the need to modernize textbooks, and better means of assimilating mathematical research into "the body of common knowledge." The conference was expected also to consider the role, organization, and operation of the MAA.

MAA President G. Baley Price convened this conference in Washington, DC on May 16-18, 1958. Subsequently, a report entitled The Washington Conference was published in the *Monthly* (65:8 (Oct. 1958) 575-586]) Among the many issues taken up by this group was a request by CUP that it be discharged as of September 1, 1958 with the expectation that it "be reorganized on a larger scale." In response, the conference passed a resolution in which it urged that the work of CUP "be continued with all possible vigor and on an expanded scale."

Consequently, on Nov. 15-16, 1958, MAA president Price convened another conference in Washington "to assist in formulating plans and policies for future work" of the now-discharged CUP was called "to re-examine the assignment of the Committee" and "to take steps to establish a new Committee with adequate funds, personnel, and program. Subsequently, a report entitled Conference on the Committee on the Undergraduate Program was published in the *Monthly* (66:3 (Mar. 1959) 213-220).

The first recommendation of this conference was to create a Committee on the Undergraduate Program in Mathematics (CUPM) with "power to delegate its various activities to subcommittees." Among the expanded priorities suggested for the new committee was to publish statements of minimal standard for teachers of mathematics in schools and colleges, as well as recommendations concerning desirable preparation for graduate programs in various fields and career areas. The new CUPM was duly appointed and held its first meeting at the end of 1958, just six weeks after the CUP Conference.

With support from the relatively new National Science Foundation, CUPM set up an office in California to support the work of the several panels and subcommittees that were established to carry out its expanded mandate. It held conferences and published newsletters to stimulate interest in various topics in twentieth century mathematics that were in high demand in the expanding post-war economy. The part of its work that had

the most widespread and lasting impact was a series of curriculum guides covering virtually every aspect of

undergraduate mathematics. For the most part, these guides were small stapled pamphlets, typed rather than typeset, that were distributed without charge using funds from the NSF grant.

The pressures on undergraduate mathematics to which CUPM was responding in these early post-Sputnik years were part of a broader pattern of crisis in mathematics from primary school through research universities. In 1963 a group of leading mathematicians undertook a serious examination of the school curriculum for the purpose of establishing long-term goals for K-12 mathematics education. Although the report of this "Cambridge Conference" is described by its authors as "tentative and highly provisional," it generated considerable controversy among both mathematicians and school teachers.

Shortly after the Cambridge Conference, the National Academy of Sciences convened a special committee to examine the status and needs of the mathematical community. Because of the distinctive role played by collegiate mathematics, this committee convened a special panel on undergraduate issues, chaired by John Kemeny (one of the first members of CUPM). Reports from the committee and the panel were published in two separate lengthy volumes (one at 256 pages, the other 114 pages). This was the first major national report to begin using the appellation "mathematical sciences" rather than "mathematics." (It was another 15 years before CUPM reports began using this same language.)

- Goals for School Mathematics: the Report of the Cambridge Conference on School Mathematics
 Educational Services Incorporated. Boston: Houghton Mifflin, 1963. Motivated by concern about
 "grave deficiencies" in the widespread post-war school curriculum reforms, and intended to provoke
 "general debate and bold experimentation," this report by leading mathematicians "is characterized
 by a complete impatience with the present capacities of the educational system." Deliberately
 leaping over practical problems such as teacher knowledge and Piagetian readiness to learn
 precepts, the report offers a vision of an idealized school curriculum to serve as goals for those
 working on more immediate programs of reform.
- The Mathematical Sciences: Vol. I, A Report. Committee on Support of Research in Mathematical Sciences (COSRIMS), Lipman Bers, chairman. National Research Council. Washington, DC: National Academy Press, 1965. This comprehensive survey, designed to "assess the present status and project future needs of the mathematical sciences," reports on the field (people, core, applications), education (undergraduate, graduate, specialties), and funding (federal, private, industry).
- The Mathematical Sciences: Vol II, Undergraduate Education. COSRIMS Panel on Undergraduate Education in Mathematics, John G. Kemeny, chairman. National Research Council. Washington, DC: National Academy Press, 1965. Anchored in eight case histories of diverse departments, this special supplement to the COSRIMS report focuses on the different needs of students with different career interests as well as a variety of problems associated with staffing.

As the economy changed and employment prospects for mathematically trained graduates declined in the early 1970s, the NSF changed priorities and withdrew support for the type of programs that supported CUPM. To complete the funded phase of CUPM's work, MAA published a two-volume *Compendium* containing many of its more recent reports. Now this *Compendium* has been scanned to make the reports in it available online in pdf format. (Some other CUPM reports are also online but available only through certain library systems.) The inventory on the following pages lists the CUPM curriculum guides and recommendations that were published in this NSF-funded phase of CUPM's work. The links to on-line versions of certain reports are

generally not to the original versions, but to the slightly updated versions that were subsequently published in the *Compendium*.

The Founding of CUPM 1958-1978: Part II

1960-61

- Recommendations of the Mathematical Association of America for the Training of Teachers of Mathematics. CUPM Panel on Teacher Training. *Amer. Mathematical Monthly*, 67:10 (Dec. 1960) 982-991.
- *Recommendations for the Training of Teachers of Mathematics*. CUPM Panel on Teacher Training. January 1961.
- Course Guides for Training of Teachers of Elementary School Mathematics. CUPM Panel on Teacher Training. June, 1961.
- Course Guides for the Training of Teachers of Junior High and High School Mathematics. CUPM Panel on Teacher Training. June, 1961.
- A Catalogue Survey of College Mathematics Courses. Committee on the Undergraduate Program in Mathematics (CUPM). December, 1961.
- *Mathematics and Engineering*. CUPM Panel on Mathematics for the Physical Sciences and Engineering, December, 1961.

1962-63

- Recommendations on the Undergraduate Mathematics Program for Engineers and Physicists.
 CUPM Panel on Mathematics for the Physical Sciences and Engineering. January, 1962.
 Preliminary Recommendations for an Honors Program in Mathematics. CUPM Panel on
 Pregraduate Training. June, 1962.
- The CUPM Catalog Survey. R.C. Buck. *Amer. Mathematical Monthly*, 69:4 (Oct. 1962) 304-306. Summary of the December, 1961 CUPM report.
- Pregraduate Preparation of Research Mathematicians. CUPM Panel on Pregraduate Training. Committee on the Undergraduate Program in Mathematics, May, 1963. "Idealized" recommendations for motivated and well-prepared students, followed by detailed course outlines for two years of "vector space calculus" followed by advanced courses in algebra, analysis, geometry, topology, and probability, with optional excursions into mathematical physics, game theory, mathematical logic, statistics, and number theory.
- Training of Teachers of Elementary School Mathematics. B. Hardgrove. Amer. Mathematical Monthly, 70:8 (Oct. 1963) 870-877.

1964-65

• Tentative Recommendations for the Undergraduate Mathematics Program for Students in the Biological, Management, and Social Sciences. CUPM Panel on Mathematics for the Biological,

Management, and Social Sciences. January, 1964.

- *Recommendations on the Undergraduate Mathematics Program for Work in Computing.* CUPM Panel on Computing. May, 1964.
- Pregraduate Training in Mathematics--A Report of a CUPM Panel. A.B. Willcox. *Amer. Mathematical Monthly*, 71:10 (Dec. 1964) 1117-1129.
- A General Curriculum in Mathematics for Colleges. Committee on the Undergraduate Program in Mathematics, W. L. Duren, Jr., chairman. Washington DC: Mathematical Association of America, 1965. Subtitled "A Report to the Mathematical Association of America," this seminal "GCMC" report proposed "an economical list" of courses which can be taught by a faculty of four and from which a "multiplicity of individual student programs can be constructed."
- Basic Library List. CUPM. January, 1965. One of the most favorably received projects of CUPM, this list comprises a nucleus of library holdings designed "to serve the essential needs of students and faculty in four-year colleges."

• *Teacher Training Supplement to Basic Library List.* CUPM Panel on Teacher Training. April, 1965. • A General Curriculum in Mathematics for Colleges. W.L. Duren, Jr. *Amer. Mathematical Monthly*, 72:8 (Oct. 1965) 825-831. A summary of the full 76 pp. GCMC report.

- Preparation for Graduate Study in Mathematics. CUPM Panel on Pregraduate Training, November, 1965. "Interim guidance" taking into account (as the Panel's idealized 1963 report does not) students' "possible deficiencies in preparation or tardiness in selecting a goal," "inadequacies of staff", or "lack of suitable textbooks." Building on the GCMC report, the Panel here firms up the pattern of one year of real analysis and one year of abstract algebra which came to dominate mathematics majors for the next two decades.
- *Mathematics Text Materials for the Undergraduate Preparation of Elementary School Teachers.* CUPM Panel on Teacher Training. November, 1965.
- Recommendations on the Undergraduate Mathematics Program for Engineers and Physicists. CUPM Panel on Mathematics for the Physical Sciences and Engineering. 1965. Slightly revised reprint of the 1962 original.
- *The Pregraduate Preparation of Research Mathematicians*. CUPM. 1965. A reprint of the 1963 report with some added commentary to clarify misunderstandings about the Panel's assumptions.

1966-67

- Calculus and the Computer Revolution. Richard W. Hamming. Berkeley, CA: Committee on the Undergraduate Program in Mathematics, 1966.
- *The Numerical Integration of Ordinary Differential Equations*. Thomas E. Hull. Berkeley, CA: Committee on the Undergraduate Program in Mathematics, 1966.
- A Curriculum in Applied Mathematics, 1966. CUPM ad hoc Subcommittee on Applied Mathematics, R. D. Anderson, chairman. Berkeley, CA: Committee on the Undergraduate Program in Mathematics, 1966.

 Mathematical Engineering—A Five-Year Program. CUPM Panel on Mathematics for the Physical Sciences and Engineering, October, 1966. Motivated by the sophisticated mathematical analysis required for America's space program, this report—based on extensive conversations with leaders of

NASA and the space industry—proposed a five-year program to train "mathematical engineers." Includes options for concentrations in three areas of special importance to the space program:

operations research, orbital mechanics, and control theory.

- *Recommendations for the Training of Teachers of Mathematics*. CUPM Panel on Teacher Training. December, 1966. Minor revisions in the 1961 report of the panel.
- CUPM, The History of an Idea. W.L. Duren, Jr. Amer. Mathematical Monthly, 74:1 Part 2 (Jan. 1967) 23-37. One of several historical studies published in the Monthly's Fiftieth Anniversary supplement. After reviewing the history of collegiate mathematics in the United States in the first half of the twentieth century, Duren gives a detailed account of the origins and history of CUP, its transformations into CUPM, and the work of its major panels. An appendix provides information on the members of CUPM and its panels.
- Recommendations on the Undergraduate Mathematics Program for Engineers and Physicists (Revised). CUPM Panel on Mathematics for the Physical Sciences and Engineering. January, 1967. A revision of the 1962 and 1965 versions.
- Qualifications for a College Faculty in Mathematics. CUPM ad hoc Committee on the Qualifications of College Teachers of Mathematics. January, 1967. Written at a time of major shortage of Ph.D graduates in mathematics, this report outlines the post-B.A, pre-Ph.D. preparation required to teach courses proposed in CUPM's GCMC report, as well as the distribution of training within a department that would be required to teach the full range of these GCMC courses. The report stresses that "continued intellectual and professional growth is essential to continued competence as a teacher. One needs to move forward in order not to fall behind."

1968-69

- Course Guides for the Training of Teachers of Elementary School Mathematics, 1968. CUPM Panel on Teacher Training. 1968. A minor revision of the influential 1963 Course Guides report from the Panel on Teacher Training serving as an intermediate step to a more thorough revision.
- A Beginning Graduate Program in Mathematics for Prospective Teachers of Undergraduates. CUPM Graduate Task Force. February, 1969. Based on data that the bulk of undergraduate mathematics teaching is done by individuals without a Ph. D., this report offers suggestions for arranging the early years of the graduate program in mathematics to provide both an appropriate background for effective college teaching and the opportunity for unretarded progress towards the Ph.D. (In retrospect, we note that the undersupply of Ph.D.s changed dramatically in just a few years after this report was written.)
- Qualifications for Teaching University-Parallel Mathematics Courses in Two Year Colleges. CUPM ad hoc Committee on the Qualifications for a Two-Year College Faculty in Mathematics. August, 1969. At a time of rapid increase in transfer enrollments in two-year colleges, the degree "most commonly held by mathematics teachers in these colleges"—the master's—is, according to this committee, "of such varying quality that it is scarcely useful as a measure of qualification for appointment, promotion, or tenure." Accordingly, this report offers recommendations for the substance of mathematical training required to teach the first part of the GCMC program "which are independent of degrees held or of credit hours earned."
- A Transfer Curriculum in Mathematics for Two Year Colleges. CUPM Panel on Mathematics in Two-Year Colleges. January, 1969. A response by CUPM to the extraordinary growth in number and diversity of two-year colleges ("new institutions are being added at the rate of one each week") with a corresponding broad range of academic backgrounds among newly recruited faculty. Although

these institutions offer a variety of programs, this CUPM report addresses only the transfer curriculum since it is "offered at almost all two-year colleges [and] local variations are least in this area."

The Founding of CUPM 1958-1978: Part III

1970-71

- Recommendations for the Undergraduate Mathematics Program for Students in the Life Sciences. An Interim Report of the Panel on Mathematics for the Life Sciences, September, 1970. Following extensive consultation with biologists and other life scientists, this panel recommended a core of mathematics for all undergraduate majors in the life sciences and outlined how this core could be related to courses suggested by CUPM in its 1965 GCMC report.
- A Course in Basic Mathematics for Colleges. CUPM. January, 1971. Recommends a substantial change in the increasingly enrolled college mathematics courses below the level of college algebra since "this second exposure is [often] no more successful than the first." In order to provide "enough mathematical literacy for adequate participation in the daily life of our present society," this report urges that flow-charting, algorithmic, and computer-related ideas be introduced early and used throughout a flexible, laboratory-based course.
- A Basic Library List for Two-Year Colleges. CUPM. January, 1971.
- Preparation for Graduate Work in Statistics. CUPM Panel on Statistics. May, 1971. Citing the 1965 COSRIMS report of the National Academy of Sciences, this report notes that "both computer science and statistics have dual sources of identity and intellectual force, only one of which is mathematical." It goes on to recommend (for colleges without separate programs in statistics) a mixed curriculum including probability, statistics, mathematics, and computing as appropriate preparation for graduate training in statistics.
- Recommendations for an Undergraduate Program in Computational Mathematics. CUPM Panel on Computing. May, 1971. In contrast to the Panel's 1964 report concerning preparation of students planning careers in computer science, this report addresses the needs of future mathematicians who will be expected to exhibit expertise in computational mathematics and scientific computing. Includes course descriptions as well as observations about staff qualifications and facility requirements.
- A Course in Basic Mathematics for Colleges. *Amer. Mathematical Monthly*, 78:6 (June-July 1971) 703. Brief announcement of the CUPM report that was issued in January, 1971.
- Recommendation on Course Content for the Training of Teachers of Mathematics. CUPM Panel on Teacher Training. August, 1971. Revision of the panel's 1961 Recommendations and Course *Guides* for teachers of elementary and secondary mathematics.
- *Report of a CUPM Conference on Computational Mathematics*. Committee on the Undergraduate Program in Mathematics. 1971.

1972-73

- Commentary on "A General Curriculum in Mathematics for Colleges". Ad hoc Committee on the Revision of GCMC. Committee on the Undergraduate Program in Mathematics. January, 1972. An analysis of how three major issues are impacting mathematics departments as they seek to implement the recommendations of the 1965 GCMC report: "the evolving nature of mathematics curricula" (e.g., mathematics vs. mathematical sciences); "the service functions of mathematics" (e.g., the demand for diverse tracks); and "the initial placement of students" (e.g., the increased variance in mathematical preparation of high school graduates). Rather than attempting a "definitive restatement" of GCMC, this *Commentary* offers recommendations for "the half-dozen courses that include a substantial part of the mathematics enrollment in almost all colleges."
- Applied Mathematics in the Undergraduate Curriculum. CUPM Panel on Applied Mathematics. January, 1972. Recommendations for enhancing the role of applied mathematics in response to "new mathematical ideas and new ways of using mathematics" associated with the increased use of mathematical methods in the social and life sciences "and even in some areas of the humanities."
- Suggestions on the Teaching of College Mathematics. CUPM Panel on College Teacher Preparation, Donald W. Bushaw, chairman. January, 1972. Largely a synthesis of hints and handouts provided by university graduate departments to their teaching assistants. Includes a special Foreword by Peter J. Hilton on the importance of effective mathematics teaching.
- Introductory Statistics Without Calculus. CUPM Panel on Statistics. June, 1972. Analysis, advice, and recommendations to improve a course that is taught by many different departments but is met with widespread dissatisfaction by statisticians because, among other ills, the courses "fail to provide intellectual stimulation" or "insight into statistical concepts."
- Recommendations on Undergraduate Mathematics Courses Involving Computing. Panel on the Impact of Computing on Mathematics Courses. October, 1972. The first attempt by CUPM to address the role of computers in the basic mathematics curriculum where a consensus "has not yet evolved." The report offers ideas for experimenting with various proposed changes in order, for example, to help students learn "constructive methods for solving problems and means to ascertain the efficiency as well as the correctness of these methods."
- Some Illustrative Examples of the Use of Undergraduate Mathematics in the Social Sciences. CUPM, 1973. A joint NSF-funded project of CUPM with the Mathematical Social Sciences Board.

Compendium of CUPM Recommendations - Part I

When NSF Funding Ended, CUPM Gathered Its Recommendations into a Two-volume Compendium

At the end of CUPM's NSF grant, the MAA published in two volumes *A Compendium of CUPM Recommendations*, subtitled "Studies, Discussions, and Recommendations by the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America." This *Compendium* contained selected reports published by CUPM since 1965 arranged by topic in seven sections, each preceded by a preface. What follows below are slight adaptations of the prefaces to each section followed by links to the contents of that section.

Compendium Preface

The Committee on the Undergraduate Program in Mathematics (CUPM) was established as a standing committee of the Mathematical Association of America (MAA) in 1959. (A detailed history of CUPM can be found in CUPM, The History of an Idea by W. L. Duren which appeared in the *American Mathematical Monthly*, 74:1 Part 2 (Jan. 1967) 23-37.) With financial assistance from the National Science Foundation, CUPM in 1960 began to engage in several projects and activities related to improvement in the undergraduate curriculum. These projects often involved the publication of reports, which were widely disseminated throughout the mathematical community and were available from the CUPM Central Office upon request. Since a change in the funding policy of the United States government makes the continuing production and free distribution of such reports extremely unlikely, the MAA has decided to publish in permanent form the most recent versions of many of the CUPM recommendations so that these reports may continue to be readily available to the mathematical community and may conveniently be kept on the reference shelves of mathematics libraries.

This *Compendium* is published in two volumes, each of which has been divided into sections according to the category of reports contained therein. These CUPM documents were produced by the cooperative efforts of literally several hundred mathematicians in the United States and Canada. The reports are reprinted here in essentially their original form; there are a few editorial comments which serve to update or cross-reference some of the materials. The editorial work for the *Compendium* was started by William E. Mastrocola during his term as Director of CUPM and completed after his return to Colgate University. He was assisted in the early stages by Andrew Sterrett and Paul Knopp, Executive Directors of CUPM during 1972 and 1973. Preparation of the final manuscript for the printer was the joint work of William E. Mastrocola and Katherine B. Magann. The considerable efforts of these individuals is deserving of special recognition.

• *Basic Library List. CUPM Compendium*, Vol. I, p. 1-32. Reprint of 1965 CUPM Report. • Commentary on A General Curriculum in Mathematics for Colleges. *CUPM Compendium*, Vol. I, p. 33-97. Slightly revised reprint of a 1972 CUPM Report.

Training of Teachers of Mathematics

CUPM's interest in the training of mathematics teachers has pervaded its activities throughout the Committee's existence. The Panel on Teacher Training, one of the original four panels, began its work at a time when mathematics instruction in elementary and secondary schools was undergoing significant changes. Throughout the years since its original report was issued, the Panel's recommendations and ongoing activities have had a profound influence on the education of elementary and secondary school teachers.

The 1961 *Recommendations for the Training of Teachers of Mathematics* identified five levels of mathematics teachers:

- I. Teachers of elementary school mathematics-grades K through 6;
- II. Teachers of the elements of algebra and geometry;
- III. Teachers of high school mathematics;
- IV. Teachers of the elements of calculus, linear algebra, probability, etc.;
- V. Teachers of college mathematics.

To complement the 1961 recommendations, CUPM also published *Course Guides for the Training of Teachers* of *Elementary School Mathematics* and *Course Guides for the Training of Teachers of Junior High School and High School Mathematics*. When it was proposed, the Level I curriculum received widespread attention and

approval. It was approved formally by the Mathematical Association of America and it was endorsed by three conferences held by the National Association of State Directors of Teacher Education and Certification (NASDTEC) and the American Association for the Advancement of Science (AAAS). It formed a part of the *Guidelines for Science and Mathematics in the Preparation Program of Elementary School Teachers*, published by NASDTEC-AAAS in 1963.

In the years 1962-66 CUPM made an intensive effort to explain its proposed Level I program to that part of the educational community especially concerned with the mathematics preparation of elementary school teachers. Forty-one conferences were held for this purpose. Participants in these conferences, who came from all fifty states, represented college mathematics departments and education departments, state departments of education, and the school systems. The details of CUPM proposals were discussed and an effort was made to identify the realistic problems of implementation of the recommendations. As a result of these conferences and of other forces for change, there was a marked increase in the level of mathematics training required for the elementary teacher.

Level II and III conferences similar to those held for Level I were deemed unnecessary because the Level II and III guidelines had apparently been accepted by the teaching community through distribution of the recommendations and course guides. One indication of this acceptance has been the publication of numerous textbooks whose prefaces claim adherence to the CUPM guidelines. Throughout the decade of the 1960's, CUPM continued to expend considerable effort on the problems associated with the preparation of teachers. Minor revisions of the original recommendations were produced in 1966, and the course guides for Level I were similarly revised in 1968.

In 1965 CUPM published *A General Curriculum in Mathematics for Colleges* (GCMC) as a model for a mathematics curriculum in a small college. GCMC became a standard reference in other CUPM documents. The shortage of mathematicians, already severe by the late 1950's, had seriously impaired the ability of many colleges to implement CUPM recommendations, including GCMC. Qualified new faculty members were extremely difficult to obtain, and many established teachers were so overloaded with teaching responsibilities that they could not keep abreast of developments in their field.

By 1965 the time was obviously ripe for CUPM to see what could be done to alleviate this problem. An *ad hoc* Committee on the Qualifications of College Teachers of Mathematics was appointed to study and report on the proper academic qualifications for teaching the GCMC courses. Simultaneously, CUPM established a Panel on College Teacher Preparation and instructed it to study a number of related topics: existing programs for the preservice and inservice training of college teachers, opportunities for support of college teacher programs by government and foundations, the supervision and training of teaching assistants, supply and demand data, etc.

In 1967 the Qualifications Committee issued its report, *Qualifications for a College Faculty in Mathematics*. The report identifies four possible components in the formal education of college teachers and describes teaching duties suitable for individuals with academic attainment equivalent to a given component. It also makes suggestions concerning the composition of a small undergraduate department.

Immediately upon publication of the qualifications report, the Panel on College Teacher Preparation fell heir to several tasks. One of these was the responsibility for a series of regional conferences designed to bring together

mathematicians and college administrators to discuss some of the issues raised by the report. Another was the

task of preparing a detailed description of a graduate program modeled after the "first graduate component" defined in the qualifications report. This latter project was undertaken by the Graduate Task Force, a group with membership drawn from the Panel and from CUPM. Its report, *A Beginning Graduate Program in Mathematics for Prospective Teachers of Undergraduates*, was issued in 1969.

Meanwhile, other members of the Panel conducted a study on the supervision and training of teaching assistants in mathematics. Their findings were reported in a newsletter published in 1968. The need for a "companion volume" for the qualifications report was established when the Panel on Mathematics in Two-Year Colleges issued its 1969 report *A Transfer Curriculum in Mathematics for Two-Year Colleges*. CUPM felt it was necessary to describe the qualifications for persons to teach the courses in the Transfer Curriculum, and for this purpose it appointed an ad hoc Committee on Qualifications for a Two-Year College Faculty in Mathematics. This group's recommendations are given in the document *Qualifications for Teaching University-Parallel Mathematics Courses in Two-Year Colleges*, published in 1969.

Publication of the several reports mentioned in the preceding paragraphs completed CUPM's original plan of providing course guides for each of the five teaching levels defined in 1961. By 1967, however, the pressure for further change was already beginning to be felt. A minor revision (1968) of the Level I course guides contained the statement:

The five years that have elapsed since the preparation of the Course Guides have seen widespread adoption of the ideas of the new elementary school curricula, not only of the work of such experimental or quasi-experimental groups as the School Mathematics Study Group (SMSG) or the University of Illinois Curriculum Study in Mathematics (UICSM), but also of many new commercial textbook series which incorporate such ideas. In addition, there have been attempts to influence the future direction of elementary school mathematics by such groups as the Cambridge Conference. In the near future, the Panel believes, it will be necessary to examine our courses to take account of these developments. We hope in the next couple of years to begin the sort of detailed, intellectual study of current trends in the curriculum and of predictions of the future which will be necessary in order to prepare teachers for the school mathematics of the next twenty years.

During the years 1968-72 the Panel on Teacher Training continued this promised study. It sought to understand current trends and future possibilities through a variety of means:

- in the spring of 1968 it sponsored a conference, "New Directions in Mathematics," to obtain the views and advice of a large number of mathematicians and educators;
- it followed the deliberations of the CUPM Panel on Computing;
- it followed with interest, and contributed to, continuing discussion on pedagogy, the changing attitudes toward experimentation in mathematics education, and the role of mathematics in society today;
- and, finally, the Panel met with representatives of the National Council of Teachers of Mathematics, the American Association for the Advancement of Science, and the National Association of State Directors of Teacher Education and Certification, and maintained contact with national curriculum planning groups.

The Panel concluded from this study that a revision of the CUPM recommendations and course guides for Levels I, II, and III was indeed required. Its 1971 report, *Recommendations on Course Content for the Training of Teachers of Mathematics*, was a result of that decision.

During the early seventies the Panel on College Teacher Preparation continued its interest in the role and preparation of teaching assistants. A 1972 newsletter, "New Methods for Teaching Elementary Courses and for the Orientation of Teaching Assistants," contains a statement by the Panel on teaching experience as part of Ph.D. programs. In 1972 the Panel also issued a booklet entitled *Suggestions on the Teaching of College Mathematics* whose purpose was to disseminate some ideas about practices that are believed to have contributed to successful teaching of mathematics in colleges and universities.

- Qualifications for a College Faculty in Mathematics. *CUPM Compendium*, Vol. I, p. 102-112. Slightly revised reprint of a 1969 CUPM Report.
- A Beginning Graduate Program in Mathematics for Prospective Teachers of Undergraduates. *CUPM Compendium*, Vol. I, p. 113-145. Slightly revised reprint of a 1969 CUPM Report.
 Recommendation on Course Content for the Training of Teachers of Mathematics. *CUPM Compendium*, Vol. I, p. 158-202. Slightly revised reprint of a 1971 CUPM Report.

Two Year Colleges and Basic Mathematics

The Panel on Mathematics in Two-Year Colleges was formed in 1966 following some preliminary study of the need and potential in this area for the kind of activities which CUPM had successfully pursued in other areas. The members of the Panel were chosen from two-year colleges, four-year colleges, and universities so that extensive experience in various phases of education would be available. The Panel initially sponsored a series of meetings at which representatives of a wide spectrum of two-year colleges provided much detailed information about local variations, supplementing the Panel's studies of the national scene. The Panel also participated in several other activities related to the problem, such as meetings of the National Science Foundation Intercommission Panel on Two-Year Colleges, meetings of various organizations of two-year college mathematics teachers, individual visits to institutions, and a wealth of personal contacts.

During this study phase the Panel was divided into subpanels concentrating on three topics: mathematics for general education, mathematics for technical-occupational programs, and mathematics for four-year college transfer programs (in all disciplines). Many two-year college teachers who consulted with the Panel expressed the opinion that guidance was most needed on the first two topics. However, it became increasingly clear as the study progressed that considerable overlap existed in the problems in these three areas and that an initial concentration on the third topic was most natural, both logically and from the viewpoint of CUPM's customary methods of operation. Thus, the Panel decided to concentrate its initial efforts on the construction of a program for university-parallel mathematics courses in two-year colleges. Its report, *A Transfer Curriculum in Mathematics for Two-Year Colleges*, was issued in 1969.

Concurrent with the decision of the Panel to restrict itself to the university-parallel curriculum, CUPM appointed an *ad hoc* Committee on Qualifications for a Two-Year College Faculty in Mathematics, whose membership overlapped that of the Panel. The report of this Committee, which appears in this Compendium's section on Training of Teachers, discusses the qualifications of teachers of university-parallel mathematics courses and makes some general remarks concerning two-year college mathematics faculties.

The Transfer Curriculum report is essentially an adaptation of the first part of *A General Curriculum in Mathematics for Colleges* to the particular circumstances of those students in two-year colleges who intend to transfer to a four-year institution. That report intentionally deferred the consideration of lower-level or non-university-parallel courses as a matter for further study. In 1970 CUPM appointed a Panel on Basic Mathematics to consider the first of these two areas: courses at a level below that of Mathematics A in the Transfer Curriculum. Among the members of this new Panel were persons from the Two-Year College Panel and representatives from developing institutions. The Panel felt that it would be possible to replace many of these courses by a single flexible course which involved a mathematics laboratory and was innovative in its approach. Its recommendations, together with an outline and commentary on the proposed course, appear in the 1971 publication *A Course in Basic Mathematics for Colleges*.

In 1971 CUPM issued *A Basic Library List for Two-Year Colleges*. This list was compiled by an *ad hoc* committee, with the assistance of many teachers from two-year colleges, four-year colleges, and universities.

Having offered suggestions for the improvement of university-parallel and basic mathematics programs, CUPM then turned to the much more complicated area of mathematics for technical-occupational programs in two-year colleges. A reconstituted Panel on Mathematics in Two-Year Colleges laid plans for producing materials designed to improve mathematics instruction for students in these fields. Due to lack of funds, it has not yet been possible to bring these plans to fruition.

- A Transfer Curriculum in Mathematics for Two Year Colleges. *CUPM Compendium*, Vol. I, p. 205-255. Slightly revised reprint of a 1969 CUPM Report.
- A Course in Basic Mathematics for Colleges. CUPM Compendium, Vol. I, p. 256-313. Slightly revised reprint of a 1971 CUPM Report.
- A Basic Library List for Two-Year Colleges. *CUPM Compendium*, Vol. I, p. 314-367. Slightly revised reprint of a 1971 CUPM Report.

Compendium of CUPM Recommendations - Part II

Pre-Graduate Training

The Panel on Pregraduate Training was appointed in 1959 to study the needs of, and to recommend programs for, undergraduate students who intend to study mathematics at the graduate level. The initial efforts of the Panel were concentrated upon the construction of an ideal curriculum for students of outstanding ability. Course outlines designed to lead the undergraduate rapidly toward the frontiers of mathematical research and the Ph.D., purposely overlooking local problems which might be caused by inadequate preparation at the secondary level or by lack of staff at the college level, appeared in the 1963 publication *Pregraduate Preparation of Research Mathematicians*.

Despite many misunderstandings regarding its assumptions and intent, this report served effectively as a basis for discussion and planning at many institutions. It was reprinted in 1965, together with some additional comments on constructive use of the booklet and admonitions to the effect that misinterpretation of the spirit of the outlines might result from a lack of knowledge of the Panel's basic assumptions and objectives. Perhaps the

report's chief value lies in showing what is regarded as ideal preparation for graduate study in pure mathematics by a very distinguished group of mathematicians.

Having completed its work on an ideal undergraduate program for the future research mathematician, the Panel turned to the urgent practical task of recommending specific undergraduate curricula for colleges which would be unable, for any of a variety of reasons, to achieve quickly the goals of its original report. These recommendations were drawn up after consultation with representatives of about 25 of the leading graduate mathematics departments. They appear in the 1965 document *Preparation for Graduate Study in Mathematics*, together with outlines for upper-division courses in Abstract Algebra and Real Analysis.

- Pregraduate Preparation of Research Mathematicians. *CUPM Compendium*, Vol. I, p. 369-446. Reprint of 1963 CUPM Report.
- Preparation for Graduate Study in Mathematics. *CUPM Compendium*, Vol. I, p. 447-457. Slightly revised reprint of a 1965 CUPM Report.

Statistics

In 1968 CUPM appointed a Panel on Statistics for the purpose of providing guidance to departments of mathematics at smaller colleges and universities on instruction in statistics. Two concerns of general interest were identified for study by the Panel: a program to prepare students for graduate study in statistics and a basic service course in statistics for students who have not studied calculus. The Panel pointed out that these two topics represent curricular extremes for statistics instruction in most undergraduate programs and that many students' program of study will lie between these extremes.

The Panel's first report, *Preparation for Graduate Work in Statistics*, was issued in 1971. This document describes the type of training and experiences which undergraduates contemplating graduate study in statistics ought to have. It outlines a basic one-year course in probability and statistics, indicates those mathematics courses which are valuable for pregraduate preparation in statistics, and comments on computer requirements and experience with data.

The Panel's second project involved a study of the introductory, non-calculus statistics courses which are offered by practically every college and taken by students in a wide variety of fields. Prompted by the fact that many of the existing courses are unsatisfactory for a variety of reasons, the Panel developed a set of objectives for such a course and made concrete suggestions for realizing the objectives. A detailed list of topics for a conventional course in introductory statistics, as well as some suggested alternate approaches, appears in the 1972 publication *Introductory Statistics Without Calculus*.

- Preparation for Graduate Work in Statistics. *CUPM Compendium*, Vol. II, p. 459-471. Slightly revised reprint of a 1971 CUPM Report.
- Introductory Statistics Without Calculus. *CUPM Compendium*, Vol. II, p. 472-525. Slightly revised reprint of a 1972 CUPM Report.

Computing

During the decade beginning in 1962, CUPM made a continuing effort to advise college mathematics departments on curricular matters related to the tremendous growth in the use of the computer and the

pervading influence which the computer has come to exert on society. Initial steps in this direction were taken by

the Panel on Physical Sciences and Engineering, which issued its *Recommendations on the Undergraduate Mathematics Program for Work in Computing* in 1964. Taking account of the significant changes which had recently occurred in the relationship of mathematics to computing and to computing machines, the Panel proposed a program designed to prepare students whose careers were likely to be intimately connected with high-speed computing. The program included reference to three types of courses: (1) mathematics courses of a general nature which should be available for the prospective specialists in computer science; (2) technical courses in computer science; and (3) an introductory course in computer science.

Two years later CUPM commissioned R. W. Hamming of Bell Telephone Laboratories, Inc., to prepare a monograph on *Calculus and the Computer Revolution*. Published in 1966, this book describes and illustrates briefly some aspects of computing as they are related to the beginning calculus course.

A task force was appointed in 1966 for the purpose of advising CUPM on a future course of action with regard to computing. The task force suggested the creation of a Panel on Computing, which would work closely with various computing organizations and would have several charges related to the impact of the computer on mathematics education. Such a panel was appointed in 1967. One of the Panel's projects was to gather and disseminate information regarding the use of computers in introductory calculus courses. A newsletter entitled *Calculus With Computers*, issued in 1969, contained general observations and summaries of statements from various institutions that had instituted computer-oriented calculus courses.

The Panel's primary aim was to develop a systematic approach to the impact of computers on undergraduate mathematics programs, rather than to address itself to the training of computer scientists *per se*. (The latter topic had already been considered by the Association for Computing Machinery in its report *Curriculum 68—Recommendations for Academic Programs in Computer Science*.) The Panel formulated a specific undergraduate program in computational mathematics, combining courses in mathematics, computer science, and computational mathematics—complete with course outlines and suggestions for implementation. This course of study is presented in the 1971 publication *Recommendations for an Undergraduate Program in Computational Mathematics*.

The main concern of this report is for the education of mathematicians who wish to know how to use and to apply computers. The report of the Panel on Computing attacked a significant problem: the need for new, innovative courses directly concerned with computational mathematics and computer science. Remaining to be considered, however, was another important question: How should the computer affect traditional mathematics courses? To study this question and related points, CUPM in 1971 appointed a Panel on the Impact of Computing on Mathematics Courses to succeed the Panel on Computing. The new Panel's investigations culminated in the publication of *Recommendations on Undergraduate Mathematics Courses Involving Computing* in 1972. This document includes outlines for lower-division courses in elementary functions, calculus, discrete mathematics, and linear algebra with stress on algorithms, approximations, model building, and the nature of the entire problem-solving process.

- Recommendations for an Undergraduate Program in Computational Mathematics. *CUPM Compendium*, Vol. II, p. 528-570. Slightly revised reprint of a 1971 CUPM Report.
- Recommendations on Undergraduate Mathematics Courses Involving Computing. *CUPM Compendium*, Vol. II, p. 571-625. Slightly revised reprint of a 1972 CUPM Report.

Applied Mathematics

The importance of applications of mathematics to other areas was recognized by CUPM early in its existence.

Among the original four panels were a Panel on Mathematics for the Physical Sciences and Engineering and a Panel on Mathematics for the Biological, Management, and Social Sciences, each charged with the task of making recommendations for the undergraduate mathematics program of students whose major interest lay in one of the stated fields.

The Panel on Physical Sciences and Engineering concentrated its efforts on the training of engineers and physicists, issuing its first report, *Recommendations on the Undergraduate Mathematics Program for Engineers and Physicists*, in 1962. The demand for this document was so great that it was necessary to have it reprinted in 1965. Significant developments which occurred during the mid-sixties prompted the Panel to revise its recommendations and issue a new report in 1967.

In the meantime this Panel had also developed CUPM's first definitive statement regarding the role of the computer in undergraduate mathematics. Its 1964 report *Recommendations on the Undergraduate Mathematics Program for Work in Computing* contained outlines for introductory and technical courses in computer science and a description of a program for mathematics majors planning to enter the field of computing. (This document is not being reproduced in this *Compendium* because it has been superseded by more recent CUPM documents discussed above in the section on Computing.)

Another document, *Mathematical Engineering--A Five-Year Program*, was issued by the Panel in 1966 to provide a means of alleviating what was then a drastic shortage of engineers having a substantial background in mathematics. Described as "a suggestion, rather than a recommendation," this report gives several outlines for options in operations research, orbit mechanics, and control theory.

The Panel on Mathematics for the Biological, Management, and Social Sciences, confronting problems which were less well defined, issued its *Tentative Recommendations for the Undergraduate Mathematics Program for Students in the Biological, Management, and Social Sciences* in 1964. Primarily concerned with the mathematics curriculum for prospective graduate students in those fields, the report was meant to serve as a basis for discussion and experimentation. As a result of several issues raised in reaction to this document, CUPM decided in 1967 to concentrate on individual disciplines and, as a first step, appointed a Panel on Mathematics in the Life Sciences, charged with making recommendations for the mathematical training of the undergraduate life science student, whether or not he goes on to graduate school. The term "life science" here referred to agriculture and renewable resources, all branches of biology, and medicine. This Panel worked closely with the Commission on Undergraduate Education in the Biological Sciences, and its investigations culminated in the publication of *Recommendations for the Undergraduate Mathematics Program for Students in the Life Sciences—An Interim Report* in 1970. Although it was anticipated that a final form of this report would eventually be issued, this project was never undertaken due to lack of funds.

Appointed in 1964, the CUPM *ad hoc* Subcommittee on Applied Mathematics was charged with suggesting appropriate undergraduate programs for students planning careers in applied mathematics. The Subcommittee's recommendations for such a program, together with suggestions for implementation and course descriptions, appeared in the 1966 report *A Curriculum in Applied Mathematics*. During the years 1967-69 an Advisory Group on Applications kept CUPM informed on current developments in applied mathematics.

The extremely rapid development of applications of mathematics, particularly in fields outside the physical sciences, together with a renewed interest in applications among mathematicians, led CUPM to appoint in 1970 a Panel on Applied Mathematics, whose duty was to reconsider some of the questions which the Subcommittee had studied earlier, and to draw up new recommendations in line with the nature and methods of applied

mathematics. The Panel's suggestions, which emphasize the role of model building, are given in the 1972 report *Applied Mathematics in the Undergraduate Curriculum*. This report contains detailed outlines of three options for a course in applied mathematics, each of which utilizes the model-building approach.

- Recommendations on the Undergraduate Program for Engineers and Physicists (Revised). *CUPM Compendium*, Vol. II, p. 628-648. Slightly revised reprint of a 1967 CUPM Report.
- Mathematical Engineering--a Five year Program. *CUPM Compendium*, Vol. II, p. 649-683. Slightly revised reprint of a 1966 CUPM Report.
- Recommendations for the Undergraduate Mathematics Program for Students in the Life Sciences (Interim Report). CUPM Compendium, Vol. II, p. 684-704. Slightly revised reprint of a 1970 CUPM Report.
- Applied Mathematics in the Undergraduate Curriculum. *CUPM Compendium*, Vol. II, p. 705-756. Slightly revised reprint of a 1972 CUPM Report.

The Turbulent 1980s

Following "A Nation at Risk," Renewed Debate about Mathematics Education

Following the conclusion of the sustaining NSF grant to CUPM and publication of the *Compendium* of its work under that grant, CUPM continued as a standing committee of the MAA with one major part of its original portfolio split off into a separate standing committee: CTUM, the Committee on the Teaching of Undergraduate Mathematics. At the same time changing circumstances in school and college education created considerable ferment not least among school, college, and university mathematicians. To set the context for CUPM's subsequent work, we list here a representative sample of reports that greatly influenced this work. A few of these reports were published by MAA, most not; many benefited from extensive advice from members of MAA:

1976-77

 Overview and Analysis of School Mathematics Grades K-12. National Advisory Committee on Mathematical Education (NACOME), Shirley A. Hill, chairman. Washington, DC: Conference Board of the Mathematical Sciences, 1975. In the aftermath of controversial "new math" reforms of the previous two decades, this synthesis of practices, proposals and evidence of achievement was designed to clarify issues, to "identify the range of viable alternative practices," and to "evaluate information useful in choosing among alternatives."

• Why the Professor Can't Teach: Mathematics and the Dilemma of University Education. Morris Kline. New York, NY: St. Martin's Press, 1977. A flamboyant indictment of collegiate mathematics education that blames a wide variety of ills on over-emphasis on research at the expense of teaching. This sequel to Kline's 1973 *Why Johnny Can't Add* was highly controversial among university administrators and mathematicians but resonated with many policy leaders and the mathophobic public.

1978-79

- PRIME-80: Prospects in Mathematics Education in the 1980s. Mathematical Association of America, 1978. Proceedings of a leadership conference convened at a time when "the entire concept of post-secondary education was undergoing review" and "fantastically rapid changes in computer technology created unexpected effects in college-level mathematics." This twenty-year sequel to MAA's 1958 Washington Conference was designed "to assess the current state of collegiate mathematics and of the related mathematical needs of the nation." The first of its several recommendations urged "new efforts" to articulate the "essential mathematical skills" needed by every citizen.
- College Mathematics: Suggestions on How to Teach It. Committee on the Teaching of Undergraduate Mathematics (CTUM). Mathematical Association of America, March, 1979. The first report of the new CTUM committee that was created to replace (and elevate) former CUPM panels.
- Training Programs for Teaching Assistants in Mathematics. Committee on the Teaching of Undergraduate Mathematics (CTUM). Mathematical Association of America, 1979. A companion to CTUM's Suggestions report. A revised version was released in 1981.
- The Role of Applications in the Undergraduate Mathematics Curriculum. Ad hoc Committee on Applied Mathematics Training, National Research Council, Peter Hilton, chairman. Washington, DC: National Academy of Sciences, 1979. Suggestions for dealing with a "grave injustice" created by many university mathematics departments when they fail to provide the option of a "mathematics curriculum for the mathematically gifted youngster who plans a career in engineering, economics, epidemiology, or biology."

1980-81

• An Agenda for Action: Recommendations for School Mathematics of the 1980s. Reston, VA: The National Council of Teachers of Mathematics, 1980. Brief exposition of eight recommendations responding to concerns that in recent responses to problems in school mathematics "a carefully reasoned sense of direction ... has been lacking." Focuses on problem solving, "stringent standards" for teaching, increased options for diverse student needs, and taking "full advantage" of calculators and computers.

• *The New Liberal Arts: An Exchange of Views.* James D. Koerner, editor. New York, NY: Alfred P. Sloan Foundation, 1981. A reflection by Sloan program officer Stephen White on the implications for higher education of the new computer age ("crudely speaking, the computer is beginning to do for the mind of man what the engine did for the muscle of man") followed by a dozen responses.

• *Training Programs for Teaching Assistants in Mathematics*, revised version. Committee on the Teaching of Undergraduate Mathematics (CTUM). Mathematical Association of America, 1981. Revision of a 1979 report.

1982-83

 A Nation at Risk: The Imperative for Educational Reform. National Commission on Excellence in Education, David P. Gardner, chairman. Washington, DC: U.S. Government Printing Office, 1983.
 Famous government report warning that "a rising tide of mediocrity" in education means that the U.S. is "committing an act of unilateral educational disarmament." While stressing the importance of the "traditional" college-prep courses in high school mathematics, this report urged the development of "new, equally demanding mathematic curricula" for students "who do not plan to continue their formal education immediately."

- New Goals for Mathematical Sciences Education. Washington, DC: The Conference Board of the Mathematical Sciences (CBMS), 1983. Report of a CBMS-sponsored conference convened "to set specific, realizable goals for improving mathematical sciences education at all levels." The conference "strongly and unanimously" recommended the establishment of a national Mathematical Sciences Education Board to "carry on all activities deemed appropriate."
- Academic Preparation for College: What Students Need to Know and Be Able to Do. New York, NY: The College Board, 1983. Includes synopsis of mathematical competencies ("broad intellectual skills required in all fields of college study") as well as basic mathematical proficiencies required for college study.
- Problem Solving in the Mathematics Curriculum: A Report, Recommendations, and an Annotated Bibliography. . Committee on the Teaching of Undergraduate Mathematics (CTUM). MAA Notes No.
 1. Mathematical Association of America, 1983. "State of the art" on problem solving in the college curriculum citing available resources, and making recommendations on ways to teach it.

1984-85

- Renewing U.S.Mathematics: Critical Resource for the Future. Committee on Resources for the Mathematical Sciences, Edward E. David, chairman. National Academy Press, 1984. This influential "David Report" on the health of the mathematical sciences stressed the need for increased support at the graduate, post-doctoral, and research levels in order to sustain the rapidly growing mathematical demands of science & technology. A synopsis appeared as an editorial in Science, 224:4654 (15 June 1984) 1189.
- High Schools and the Changing Workplace: The Employer's View. Panel on Secondary School Education for the Changing Workplace, Committee on Science, Engineering and Public Policy, Richard E. Heckert, chairman. Washington, DC: National Academy Press, 1964. Focused on the 40 percent of high school graduates who enter the U.S. workforce rather than continuing to a four-year college, this influential report was one of the first to argue that such students "need virtually the same competencies as those who go on to college, but have less opportunity or time to acquire them."
- Involvement in Learning: Realizing the Potential of American Higher Education. Study Group on the Conditions of Excellence in American Higher Education. Washington, DC: National Institute of Education, October, 1984. Intended to "turn the spotlight on higher education, the level of the [education] system from which the other levels so often take their cues," this report argues for articulation and measurement of outcomes as a standard for excellence in higher education (in contrast with more common input measures and other proxies for excellence).

 Integrity in the College Curriculum: A Report to the Academic Community. Project on Redefining the Meaning and Purpose of Baccalaureate Degrees. Washington, DC: Association of American Colleges, February, 1985. Findings and recommendations in response to a perceived "decline and devaluation" of the bachelor's degree; stresses understanding of numerical data as part of the "minimum required curriculum." • Academic Preparation in Mathematics: Teaching for Transition From High School To College.. Jeremy Kilpatrick, editor. College Entrance Examination Board (CEEB), 1985. Written to provide "assistance to teachers who would like to help students enter college with the kind of preparation in mathematics they need to be successful.".

1986-87

- Undergraduate Science, Mathematics and Engineering Education, Vols. I, II. Task Committee on Undergraduate Science and Engineering Education. National Science Board, 1986. Policy recommendations for the National Science Foundation focused on strengthening undergraduate programs.
- School Mathematics in the 1990s. Geoffrey Howson and Bryan Wilson. ICMI Study Series, v. 2. Cambridge University Press, 1986. Report synthesizing international views on the rapidly evolving impact of modern technology on the aims, content, and teaching of mathematics.
- *Mathematical Sciences: A Unifying and Dynamic Resource*. Panel on Mathematical Sciences, Board on Mathematical Sciences. Washington, DC: National Academy Press, 1986. One in a series of "state of the art" reviews requested by the National Science Foundation to assess world trends, relative strengths, and competitiveness of the U.S. in rapidly evolving areas of science and technology.
- The Underachieving Curriculum: Assessing US. School Mathematics from an International Perspective. Curtis C. McKnight, et al. Champaign, IL:: Stipes Publishing Co., 1987. The U.S. National Report on the Second International Mathematics Study (SIMS).
- *Toward a Lean and Lively Calculus*. Ronald Douglas, editor. MAA Notes No. 6. Mathematical Association of America, 1987. Proceedings of the 1986 conference on calculus reform held at Tulane University.

1988-89

- "Renewing U.S. Mathematics: An Agenda to Begin the Second Century." Edward E. David. Notices of Amer. Math. Soc., 35 (Oct. 1988) 1119-1123. Summary and extension of the author's 1984 Critical Resources report.
- Calculus for a New Century: A Pump, Not a Filter. Lynn A. Steen, editor. MAA Notes No. 8. Mathematical Association of America, 1988. Proceedings of the 1987 Calculus Colloquium at the National Academy of Sciences in Washington, DC.

Reshaping College Mathematics

CUPM's Responses Include a Shift from Mathematics to Mathematical Sciences

During this same period, CUPM continued to publish reports on areas of vital importance to collegiate mathematics. First came a major updating of its earlier recommendations on the undergraduate major. Whereas formerly the focus of attention was on students intending to attend graduate school in mathematics or a

mathematically intensive field, changing demographics of undergraduates caused colleges, and hence CUPM, to develop principles for a mathematics major for students with much broader goals.

CUPM also issued reports focused on the elementary part of the curriculum where the vast majority of students encounter college mathematics. Two of these looked at the experiences of college students who take the lowest level courses in "mathematics appreciation" or courses ordinarily taught in secondary school. Three others begin to examine issues concerning the nature of introductory college mathematics in the emerging computer age—that is, continuous vs. discrete mathematics. As calculus seeped into the standard secondary school academic track, and student interest tilted towards computer science, major issues at this interface became increasingly problematic. By 1989 CUPM once again pulled together its major recent reports in a compendium entitled *Reshaping College Mathematics*. (As with the previous compendium, links are to the republished versions rather than to the originals.)

1976-79

- Case Studies in Applied Mathematics. Committee on the Undergraduate Program in Mathematics (CUPM). Mathematical Association of America, 1976. Nine case studies presented at a CUPM-sponsored conference, Most include student exercises and projects, as well as reports from their trial use.
- A Basic Library List for Four-Year Colleges, Second Edition. CUPM Subcommittee on Revision of the Basic Library List, Daniel T. Finkbeiner, chairman. MAA, 1976. An updating of the popular 1965 original version that is more than twice as large: 700 titles now vs. 300 earlier. (In both editions, the recommended minimal core library included approximately half the total number of titles.)
- A Basic Library List for Two-Year Colleges, Second Edition. CUPM Subcommittee on Basic Library Lists, Daniel T. Finkbeiner, chairman. MAA, 1980. Updating of the 1971 original version.

1980-84

- Recommendations for a General Mathematical Sciences Program. CUPM Panel on a General Mathematical Sciences Program, Alan Tucker, chairman. Mathematical Association of America, 1981. A lengthy and substantial revision of CUPM's 1965 and 1972 *General Curriculum in Mathematics for Colleges* that refocuses upper level courses on mathematical reasoning and "mastery of mathematical tools needed for a life-long series of different jobs and continuing education." Echoing the 1965 COSRIMS report, "CUPM now believes that the undergraduate major offered by a mathematics department at most American colleges and universities should be called a Mathematical Sciences major." (As in many other instances, the link given here is not to the original, but to a 1989 updated reprint.)
- Minimal Mathematical Competencies for College Graduates. CUPM Panel on Minimal Mathematical Competencies of College Graduates, Donald Bushaw, chairman. Amer. Mathematical Monthly, 89:4 (Apr. 1982) 266-272. Reviews of pertinent literature together with surveys of scientific leaders and randomly selected mathematicians led to "bare minimum" recommendations which individual institutions should exceed "as much as local conditions allow."
- Mathematics Appreciation Courses. CUPM Panel on Mathematics Appreciation Courses, Jerome Goldstein, chairman. Amer. Mathematical Monthly, 90:1 (Jan. 1983) 44-51, C11-C20.
 Recommendations for mathematics courses taken by humanities students, "frequently as their last

formal contact with mathematics." Includes an extensive (but separately published) Bibliography.

• *Recommendations on the Mathematical Preparation of Teachers*. Committee on the Teaching of Undergraduate Mathematics. MAA Notes No. 2, Mathematical Association of America, 1983. A translation into specific course descriptions of NCTM's 1979 report *Guidelines for the Preparation of Teachers of Mathematics*.

1985-87

- *Report of the Committee on Discrete Mathematics in the First Two Years*. Mathematical Association of America, 1986. Recommendations from the committee followed by reports of six innovation projects funded by the Alfred P. Sloan foundation.
- Curriculum for Grades 11-13. Mathematical Association of America and National Council of Teachers of Mathematics, 1987. A joint report that was approved by the Boards of both MAA & NCTM. Divided into issues of "apparent consensus" and issues on which there is lack of consensus and which, therefore, require further study.
- Problems in Transition from High School to College Calculus. CUPM Panel on Calculus Articulation, Donald B. Small, chairman. *Amer. Mathematical Monthly*, 94:8 (Oct. 1987) 776-785. Slight revision of the Panel's report which surveys problems in articulation and recommends options for improvement.
- Teaching Assistants and Part-time Instructors: A Challenge. Committee on Teaching Assistants and Part-Time Instructors, Bettye Anne Case, chairman. Mathematical Association of America, 1987.
 Recommendations of a new CTUM subcommittee derived from a data-based analysis of difficulties created by the changing demographics of teaching assistants and increases in part-time faculty.
- Reshaping College Mathematics. MAA Notes No. 8. Mathematical Association of America, 1989. Reprinting of all CUPM reports published between 1981 and 1987 including the seminal 1981 *General Mathematical Sciences Program* report.

• Preface to *Reshaping College Mathematics*. Lynn A. Steen, CUPM Chair. *Reshaping*, pp. vii-x. • Mathematical Sciences. CUPM Panel on a General Mathematical Sciences Program, Alan Tucker, chairman. *Reshaping*, pp. 1-17. Reprint of Preface & Chapter I of 1981 GMSP together with a new preface reflecting on issues arising in the decade since publication of GMSP.

- Calculus. CUPM Subpanel on Calculus, Donald Kreider, chairman. *Reshaping*, pp. 19-23. Reprint of Chapter II of 1981 GMSP.
- Core Mathematics. CUPM Subpanel on Core Mathematics, Paul Campbell, chairman. *Reshaping*, pp. 25-28. Reprint of Chapter III of 1981 GMSP.
- Computer Science. CUPM Subpanel on Computer Science, chair Tucker. *Reshaping*, pp. 29-39. Reprint of Chapter IV of 1981 GMSP.

• Modeling and Operations Research. CUPM Subpanel on Modeling and Operations Research, William Lucas, chairman. *Reshaping*, pp. 41-54. Reprint of Chapter V of 1981 GMSP. • Statistics. CUPM Subpanel on Statistics, Richard Alo, chairman. *Reshaping*, pp. 55-59. Reprint of Chapter VI of 1981 GMSP.

- Discrete Mathematics. Committee on Discrete Mathematics, Martha Siegel, chairman. *Reshaping*, pp. 61-84. Slightly revised reprint of a 1986 committee report with a new preface.
- Calculus Transition: From High School to College. CUPM Panel on Calculus Articulation, Donald B. Small, chairman. *Reshaping*, pp. 85-90. Slightly revised reprint of a 1987 panel report with a new

preface.

- Curriculum for Grades 11-13. Joint MAA-NCTM Task Force on Curriculum for Grades 11-13, Joan Leitzel, chairman. *Reshaping*, pp. 91-102. Slightly revised reprint of a 1987 (unpublished) task force report.
- Minimal Mathematical Competencies for College Graduates. CUPM Panel on Minimal Mathematical Competencies of College Graduates, Donald Bushaw, chairman. *Reshaping*, pp. 103-108. Slightly revised reprint of a 1982 panel report with new preface.
- Mathematics Appreciation Courses. CUPM Panel on Mathematics Appreciation Courses, Jerome Goldstein. *Reshaping*, pp. 109-125. Reprint of 1883 panel report with new preface & appended references.

Emergence of Goals & Standards

Support and Controversy over the National Standards Movement

In 1989 several new forces emerged that began to disrupt the landscape of mathematics education. Nationally, President George H.W. Bush and Arkansas Governor Bill Clinton in his capacity as chairman of the National Governors Association convened the nation's governors for an unprecedented Education Summit to set forth national goals for education under the slogan "America 2000." One of these goals was that "by the year 2000, U.S. students will be first in the world in science and mathematics achievement."

At the same time, the National Council of Teachers of Mathematics, published the first version of proposed national standards for school mathematics. Shortly thereafter, the National Research Council produced two reports on the urgency of revitalizing the mathematical sciences—one about issues, the other about people and demographics. Here is a sample of some of the major reports from sources other than MAA that carry consequences for undergraduate mathematics:

- Curriculum and Evaluation Standards for School Mathematics. Commission for Standards for School Mathematics. National Council of Teachers of Mathematics, 1989. The first version of NCTM's Standards aimed at producing an informed electorate and mathematically literate adults. These were the first set of nation-wide standards developed for any school subject in the U.S.
- Everybody Counts: A Report to the Nation on the Future of Mathematics Education. Committee on Mathematical Sciences in the Year 2000, Mathematical Sciences Education Board (MSEB), and Board on Mathematical Sciences. National Academy Press, 1989. A synthesis of the thinking of these three boards at the National Research Council describing forces that impinge on mathematics and on education—computers, research, demography, competitiveness—and how the interaction among these forces produces a system that is peculiarly resistant to change.
- *Fifty Hours: A Core Curriculum for College Students.* Lynne V. Cheney, Chairman. Washington, DC: National Endowment for the Humanities, 1989. Proposal for 50 hours of core study for all undergraduates, including a one-year six-hour course focusing on major concepts, methods, and

applications of the mathematical sciences, including "theoretical advances from the ancient to the contemporary" and "applications in such areas as business, economics, statistics, science, and art."

- Renewing U.S. Mathematics: A Plan for the 1990s. Committee on the Mathematical Sciences: Status and Future Directions, Edward E. David, chairman. Board on Mathematical Sciences, National Research Council. Washington, DC: National Academy Press, 1990. This update of the 1984
 "David Report" on the status of the mathematical sciences concludes that, for a variety of reasons, "the academic foundations of the mathematical sciences research enterprise are as shaky now as in 1984."
- Actions for Renewing U.S. Mathematical Sciences Departments. Board on Mathematical Sciences. National Research Council, 1990. A compilation of ideas to aid university mathematics departments in designing their own improvement plans in response to the various calls to renew the mathematical sciences enterprise in the United States.
- A Challenge of Numbers: People in the Mathematical Sciences. NRC Committee on the Mathematical Sciences in the Year 2000, Bernard L. Madison & Therese A. Hart. National Academy Press, 1990.
 Documenting an urgent need for revitalization, this report from the MS 2000 project provides a comprehensive set of data describing the demographic situation in the mathematical sciences.
- *The New Liberal Arts Program: A 1990 Report.* Samuel Goldberg, editor. New York: Alfred P. Sloan Foundation, 1990. A collection of reports from projects supported by a \$20 million undertaking begun in 1980 by the Sloan Foundation to "infuse meaningful experiences" with quantitative, mathematical, and technological approaches to a wide range of liberal arts fields.
- Scholarship Reconsidered: Priorities of the Professoriate. Ernest L. Boyer. Princeton, NJ: The Carnegie Foundation for the Advancement of Teaching, 1990. An influential effort to expand the definition of scholarship in a way that would enhance the quality of college and university education. Explores four types of scholarship: of discovery, of integration, of application, and of teaching.
- America 2000: An Education Strategy. U.S. Department of Education. U.S. Government Printing office, 1991. A brief report outlining strategies for the president, congress, governors, businesses, and parents to help achieve the national education goals that emerged from the 1989 Education Summit.
- What Work Requires of Schools: A SCANS Report for America 2000. The Secretary's Commission on Achieving Necessary Skills. U.S. Department of Labor. U.S. Government Printing office, June, 1991. A report on the implications for learning of changes in the world of work created by world-wide competition and the information-age economy.

During this same period MAA's work on policy for collegiate mathematics was divided among several committees, panels, and task forces. Many were only loosely associated with CUPM. (As noted earlier, MAA subsequently set up a Council structure to group committees in related areas.) In 1992 the MAA published yet another "compendium" of reports concerning the undergraduate curriculum, this one containing both reprints and new reports from innovative email "focus groups" whose goals were to record a variety of helpful ideas rather than to make specific consensus-based recommendations.

Here is a list (with some links) of MAA publications on the undergraduate program in mathematics from this period, only a few of which emerged as reports by or of CUPM:

• A Curriculum in Flux: Mathematics at Two-Year Colleges. CUPM Subcommittee on Mathematics Curriculum at Two-Year Colleges, Ronald M. Davis, Editor. MAA Reports No. 1. Mathematical Association of America, 1989. Recommendations prepared by a joint committee of the Mathematical Association of America (MAA) and the American Mathematical Association of Two-Year Colleges (AMATYC).

- A Source Book for College Mathematics Teaching. Committee on the Teaching of Undergraduate Mathematics (CTUM), Alan H. Schoenfeld, editor. MAA Reports No. 2. Mathematical Association of America, 1989.
- How Should Mathematicians Prepare for College Teaching? Bettye Anne Case. Notices of American Mathematical Society, 36 (Dec. 1989) 1344-1346.
- Undergraduate Major in the Mathematical Sciences. CUPM Subcommittee on the Major in the Mathematical Sciences, Bettye Anne Case, chairman. Mathematical Association of America, 1991. A brief sequel to CUPM's previous major curricular report (in 1981) reflecting important practices that emerged in the 1980s and recommending new areas of emphasis.
- A Call for Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics. Committee on the Mathematical Education of Teachers (COMET), James R. C. Leitzel, Chairman. MAA Reports No. 3. Mathematical Association of America, 1989.
- Challenges for College Mathematics: An Agenda for the Next Decade. Joint Task Force on Study in Depth of the MAA and AAC (the Association of American Colleges), Lynn A. Steen, chairman. *Focus*, 10:6 (Nov.-Dec. 1990) Center Sect., p. 1-28. One of several disciplinary reports on the relation of "study in depth" to goals for the undergraduate major, assurance of students' intellectual development, and connections with other fields. Part of a sequel to *Integrity in the College Curriculum* (AAC, 1985). (A summary appeared in *Liberal Learning and the Arts and Sciences Major, Vol 2: Reports from the Fields*, AAC, 1990, pp.77-95.)

• *Guidelines for the Continuing Mathematical Education of Teachers*. Committee on the Mathematical Education of Teachers (COMET). MAA Notes No. 10. Mathematical Association of America, 1992. • *Responses to the Challenge: Keys to Improved Instruction by Teaching Assistants and Part-Time Instructors*. Committee on Teaching Assistants and Part-Time Instructors, Bettye Anne Case, Editor. MAA Notes No. 11. Mathematical Association of America, 1992.

- The Use of Calculators in the Standardized Testing of Mathematics. John Kenelly, Editor. MAA Notes No. 12. Mathematical Association of America, 1992. A joint publication with The College Board.
- *Discrete Mathematics in the First Two Years*. Anthony Ralston, Editor. MAA Notes No. 15. Mathematical Association of America, 1992.
- *Priming the Calculus Pump: Innovations and Resources*. CUPM Subcommittee on Calculus Reform and the First Two Years (CRAFTY), Thomas W. Tucker, Editor. MAA Notes No. 17 Mathematical Association of America, 1992.
- Library Recommendations for Undergraduate Mathematics. Ad hoc CUPM Subcommittee on the Basic Library List, Lynn A. Steen, chairman. MAA Reports No. 4. Mathematical Association of America, 1992. A major updating of the Basic Library List first published in 1965 and revised in 1976. This edition contains 3000 titles (as compared with 300 in 1965 and 700 in 1976); subsequent to this publication the Basic Library List was redesigned as an on-line resource.
- Two-Year College Mathematics Library Recommendations . Ad hoc CUPM Subcommittee on the Basic Library List, Lynn A. Steen, chairman. MAA Reports No. 5. Mathematical Association of America, 1992. A major updating of the Basic Library List for Two Year Colleges first published in 1971, then revised in 1980. Two much shorter variations were also published—one for high

school libraries, and other for public libraries.

- Heeding the Call for Change: Suggestions for Curricular Action. Lynn A. Steen, editor. MAA Notes No. 8. Mathematical Association of America, 1992. Reports from e-mail focus groups discussing curricular priorities in five different areas supplemented by commissioned essays on two areas of
- emerging controversy as well as reprints of some earlier reports of CUPM subcommittees: Preface to Heeding the Call for Change. Heeding, pp. vi-ix.
 - Teaching Statistics. CUPM E-mail Focus Group on Statistics, George Cobb, moderator. *Heeding*, pp. 3-34.
 - Report of Workshop on Statistical Education. Robert V. Hogg. *Heeding*, pp. 34-43. Reprint of the report of a 1990 workshop sponsored by the American Statistical Association.
 - Tomorrow's Geometry. CUPM E-mail Focus Group on Geometry, Joseph Malkevitch, moderator. *Heeding*, pp. 47-68.
 - Recommendations of a Geometry Conference. Consortium for Mathematics and its Applications (COMAP). *Heeding*, pp. 68-69. Reprinted from "Geometry's Future," COMAP, 1990.
 - Geometry: Yesterday, Today, & Tomorrow. Joseph Malkevitch. *Heeding*, pp. 69-80. Reprinted from "Geometry's Future," COMAP, 1990.
 - Environmental Mathematics. CUPM E-mail Focus Group on Environmental Mathematics, Ben Fusaro, moderator. *Heeding*, pp. 83-89.
 - Mathematicians Develop New Tools to Tackle Environmental Problems. Chronicle of Higher Education, David L. Wheeler. *Heeding*, pp. 89-92. Reprinted from the *Chronicle* of Higher Education, 38:20 (Jan 22, 1992) pp. A7, A10-A11.
 - Why Quantitative Literacy?. Donald Bushaw. *Heeding*, pp. 95-99. Reflections on the challenge of quantitative literacy that is "clearly not being met" since "about half of American colleges and universities have no general mathematics requirement for graduation."
 - Reaching for Quantitative Literacy. CUPM E-mail Focus Group on Quantitative Literacy, Linda Sons, moderator. *Heeding*, pp. 95-118.
 - Multiculturalism in Mathematics: Historical Truth or Political Correctness? Allyn Jackson. *Heeding*, pp. 121-134. A commissioned review of issues surrounding the relation of mathematics to cultural issues.
 - Assessment of Undergraduate Mathematics. CUPM E-mail Focus Group on Assessment , Bernard L. Madison. *Heeding*, pp. 137-149.

• Toward Assessment Worthy of the Liberal Arts. Grant Wiggins. *Heeding*, pp. 150-162. Reprint of a paper presented to the AAHE Assessment Forum, 1990, pp. 17-31. • Untying the Mind's Knot. Barry Cipra. *Heeding*, pp. 165-172. Commissioned review of issues surrounding research in undergraduate mathematics education.

- Communication Among Communities. Report of a November 1991 conference on research in collegiate mathematics, Nov. 1991. *Heeding*, pp. 173-181.
- Challenges for College Mathematics: An Agenda for the Next Decade. MAA-AAC Task Force on Study in Depth, Lynn A. Steen, chairman. *Heeding*, pp. 185-223. Slightly revised reprint of a 1990 MAA-AAC report on study in depth.
- The Undergraduate Major in the Mathematical Sciences. CUPM Subcommittee on the

Major in the Mathematical Sciences, Bettye Anne Case, chairman. *Heeding*, pp. 227-247. Reprint of the 1991 update of CUPM's 1981 GCMS report (which itself is reprinted in *Reshaping College Mathematics* (MAA, 1989).

Forty years after MAA's began the Committee on the Undergraduate Program in Mathematics two conferences were held to review the history and evolution of the core curriculum in collegiate mathematics. The opening essay surveyed the evolution of CUPM's recommendations; the entire volume, published in 1998, contains papers touching on those mathematics courses taken by the vast majority of U.S. college students.

- Confronting the Core Curriculum: History, Goals, Models, Challenges. Lynn Arthur Steen. Confronting the Core Curriculum: History, Goals, Models, Challenges, p. 3-13. Keynote address at the 1994 Core Curriculum conference outlining the evolution of the undergraduate mathematics curriculum since the founding of CUPM 40 years earlier.
- Core Curriculum in Context: Considering Change in the Undergraduate Mathematics Major. John A. Dossey, editor. MAA Notes No. 45. Mathematical Association of America, 1998. Papers from two conferences held at the U.S. Military Academy at West Point in 1994 and 1995 on challenges posed by changes in the first two years of undergraduate mathematics.

Beginning in the mid-1990s, reports from CUPM and other MAA committees dealing with the undergraduate curriculum were generally published on-line (and sometimes only on-line). Access to these more recent reports is available from the CUPM Web Page.

References Arranged by Topic - Part I

An Index of Links and Citations Grouped by Topic

As a reference tool for those who may be interested in certain aspects of CUPM's work, we list here the references cited above, grouped by topic. Topics are arranged in an order that moves from more general and elementary to more specialized and advanced; within topics, references are listed in chronological order. (In a few cases a citation is listed twice if it fits naturally under two different topic headers.) These reference lists include reports that bear on the agenda of CUPM whether or not they were sponsored by CUPM or its various associated subcommittees. Quite a few are studies by other groups or individuals, many of whom had close ties to CUPM or MAA. All were mentioned earlier in this document.

Since many CUPM reports have been reprinted in one of three compendia, we use these abbreviations in citations:

- Compendium:—A Compendium of CUPM Recommendations (2 Vols). Mathematical Association of America, c. 1976.
- Reshaping:-Reshaping College Mathematics. Lynn Arthur Steen, editor. MAA Notes No. 13.

Mathematical Association of America, 1989.

• *Heeding*:—Heeding the Call for Change: Suggestions for Curricular Action. Lynn A. Steen, editor. MAA Notes No. 22. Mathematical Association of America, 1992.

As above, although CUPM citations refer to the original publication, most links go to the version found in the compendia rather than to an on-line copy of the original.

Policy Issues in Education and Mathematics

- 1965. The Mathematical Sciences: Vol. I, A Report. Committee on Support of Research in Mathematical Sciences (COSRIMS), Lipman Bers, chairman. National Research Council. Washington, DC: National Academy Press, 1965.
- 1965. The Mathematical Sciences: Vol II, Undergraduate Education. COSRIMS Panel on Undergraduate Education in Mathematics, John G. Kemeny, chairman. National Research Council. Washington, DC: National Academy Press, 1965.
- 1981. *The New Liberal Arts: An Exchange of Views*. James D. Koerner, editor. New York, NY: Alfred P. Sloan Foundation, 1981.

• 1983. A Nation at Risk: The Imperative for Educational Reform. National Commission on Excellence in Education, David P. Gardner, chairman. Washington, DC: U.S. Government Printing Office, 1983. • 1984. *Involvement in Learning: Realizing the Potential of American Higher Education.* Study Group on the Conditions of Excellence in American Higher Education. Washington, DC: National Institute of Education, October, 1984.

- 1984. Renewing U.S.Mathematics: Critical Resource for the Future. Committee on Resources for the Mathematical Sciences, Edward E. David, chairman. National Academy Press, 1984. (See Synopsis, Science, 224:4654 (15 June 1984) 1189.
- 1985. Integrity in the College Curriculum: A Report to the Academic Community. Project on Redefining the Meaning and Purpose of Baccalaureate Degrees. Washington, DC: Association of American Colleges, February, 1985.
- 1986. *Mathematical Sciences: A Unifying and Dynamic Resource*. Panel on Mathematical Sciences, Board on Mathematical Sciences. Washington, DC: National Academy Press, 1986.

• 1986. Undergraduate Science, Mathematics and Engineering Education, Vols. I, II. Task Committee on Undergraduate Science and Engineering Education. National Science Board, 1986. • 1988 "Renewing U.S. Mathematics: An Agenda to Begin the Second Century." Edward E. David. Notices of Amer. Math. Soc., 35 (Oct. 1988) 1119-1123.

- 1990. A Challenge of Numbers: People in the Mathematical Sciences. NRC Committee on the Mathematical Sciences in the Year 2000, Bernard L. Madison & Therese A. Hart. National Academy Press, 1990.
- 1990. Actions for Renewing U.S. Mathematical Sciences Departments. Board on Mathematical Sciences. National Research Council, 1990.
- 1990 *Liberal Learning and the Arts and Sciences Major.* Vol 1: The Challenge of Connected Learning; Vol 2: Reports from the Fields. Association of American Colleges, 1990.

• 1990. *Renewing U.S. Mathematics: A Plan for the 1990s.* Committee on the Mathematical Sciences: Status and Future Directions, Edward E. David, chairman. Board on Mathematical Sciences, National Research Council. Washington, DC: National Academy Press, 1990.

- 1990. *Scholarship Reconsidered: Priorities of the Professoriate*. Ernest L. Boyer. Princeton, NJ: The Carnegie Foundation for the Advancement of Teaching, 1990.
- 1990. *The New Liberal Arts Program: A 1990 Report.* Samuel Goldberg, editor. New York: Alfred P. Sloan Foundation, 1990.

Reports about CUPM, its History, and Recommendations • 1953. 36th

Annual Meeting of the MAA. *Amer. Mathematical Monthly*, 60:2 (Mar. 1953) 214-218. • 1955. Report of the Committee on the Undergraduate Mathematical Program. W.L. Duren *et al. Amer. Mathematical Monthly*, 62:7 (Sep. 1955) 511-520.

- 1957. *Collected Reports of the CUP*. Committee on the Undergraduate Program, W.L. Duren, Jr., editor. University of Virginia, 1957.
- 1958. *Transition Report and Outline of Recommended Courses*. R.L. Davis. University of Virginia. 1958.

• 1958. The Washington Conference. *Amer. Mathematical Monthly*, 65:8 (Oct. 1958) 575-586. • 1959. Conference on the Committee on the Undergraduate Program *Amer. Mathematical Monthly*, 66:3 (Mar. 1959) 213-220.

- 1967. CUPM, The History of an Idea. W.L. Duren, Jr. *Amer. Mathematical Monthly*, 74:1 Part 2, Fiftieth Anniversary Issue (Jan. 1967) 23-37.
- 1976. A Compendium of CUPM Recommendations (2 Vols). Mathematical Association of America,
 c. 1976. (Subtitled "Studies, Discussions, and Recommendations by the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America.")

1989. Reshaping College Mathematics. Lynn Arthur Steen, editor. MAA Notes No. 13. Mathematical Association of America, 1989. (See Preface for an overview of CUPM reports in the 1980s.)
 1992. Heeding the Call for Change: Suggestions for Curricular Action. Lynn A. Steen, editor. MAA Notes No. 22. Mathematical Association of America, 1992.

Universal Mathematics

- 1954. *Universal Mathematics, Part I: Functions and Limits*. F.A.I. Bowers, Jr., R.N. Bradt, C.E. Capel, W.L. Duren, Jr., G.B. Price, W.R. Scott. University of Kansas, 1954.
- 1955. *Universal Mathematics, Part II: Elementary Mathematics of Sets*. R.N. Bradt, C.E. Capel, W.L. Duren, Jr., J.G. Kemeny, E.J. McShane, D.R. Morrison, G.B. Price, A.L. Putnam, W.R. Scott, & A.W. Tucker. Tulane University 1955.
- 1956. Review by Herbert (H.P.) Evans of *Universal Mathematics, Part I. Amer. Mathematical Monthly*, 63 (1956) 196-199.
- 1956. Report by W. L. Duren on a trial of *Universal Mathematics, Part I. Amer. Mathematical Monthly*, 63 (1956) 199-202.
- 1957. A Freshman Honors Course in Calculus and Analytic Geometry Taught at Princeton University. Emil Artin & George B. Seligman. Committee on the Undergraduate Program, Mathematical Association of America, 1957; University of Buffalo, 1958.
- 1958. Review of A Freshman Honors Course in Calculus and Analytic Geometry. Amer. Mathematical Monthly, 65 (1958) 538-539.
- 1958. Elementary Mathematics of Sets with Applications. R.N. Brandt & R.L. Davis. Mathematical

Association of America, 1958.

- 1958. Modern Mathematical Methods and Models, Vol. 1: Multicomponent Methods. Dartmouth College Writing Group., 1958.
- 1959. Review by Richard Johnson of *Modern Mathematical Methods and Models, Vol. 1. Amer. Mathematical Monthly*, 66 (March, 1959) 246-247.

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