Carl B. Allendoerfer Awards

The Carl B. Allendoerfer Award, established in 1976, is made to authors of expository articles published in *Mathematics Magazine*. Carl B. Allendoerfer, a distinguished mathematician at the University of Washington, served as president of the Mathematical Association of America, 1959–60.

Alissa S. Crans and Glen T. Whitney

Integral Tiling Pentagons, *Mathematics Magazine,* 96:2, 130–140.
doi.org/10.1080/0025570X.2023.2176101

For nearly a hundred years, mathematicians—professionals and amateurs alike—have been on a quest to find all convex pentagons that tile the plane. In their paper, the authors gift readers of *Mathematics Magazine* in several ways: they give a lively recounting of the quest, acknowledging the sometimes decades-long gaps between advances; they concisely summarize the status of the problem (namely, that 15 families of convex pentagon tilings were discovered between 1918–2015; and, in 2017, mathematician Michael Rao announced that he had computationally verified that the list of 15 families is complete); and as per the title of their paper, they completely solve the integral tiling pentagons problem, providing "exact criteria to determine whether a given 5-tuple of natural numbers can occur as the side lengths of a pentagon within each of the 15 families."

The writing and the organization of the paper are outstanding. Proofs are accessible, relying, as the authors note, on classical number theory and properties of continuity and convexity. Geometric diagrams that show side-angle relationships in pentagons from different families illuminate the proof, as do tables of integer side lengths and "smallest examples" that are arranged by family. Readers finish with clear take-aways: (1) "The 15 families are characterized by relations among their sides and angles." (2) Families 14 and 15 do not admit even rational side lengths. (3) The other 13 families "have infinitely many dissimilar, integer-sided representatives." (4) Families 1–13 form two groups: a group of five families whose solutions involve Diophantine equations, and a group of eight families, where the integer constraint plays only a minor role and whose solutions draw upon convexity.

Of special mention, this paper offers an example of how to engage and inspire the public with mathematics. On being invited to speak at a United Nations conference on educating girls, the authors relate how they planned to engage audience members in experiencing and discovering mathematics by providing them with store-bought, integer-length rods and inviting them to try their hand at constructing integral pentagonal tilings. This sharing is another gift to readers, adding to the rich contributions of the paper.

Responses

We are thrilled and honored to have our work recognized with the Allendoerfer Award. Jason Rosenhouse and the reviewers improved the quality of our exposition, for which we are grateful, and they deserve their share of the credit. We hope that the outreach leading to our article could help encourage others to share their love of mathematics with broad, diverse audiences, especially in hands-on ways. Who knows, you might also find a whole new problem to work on, as we did! "Integral Tiling Pentagons" only includes the details for fewer than half of the families, leaving ample opportunity for interested readers and their students to explore the remaining ones. Although of similar flavor, each has its own unique charms. In the end, we're just delighted to have contributed a tiny epilogue to the hundred-year history of this wonderful tiling problem.

Biographical Sketches

As a professor of mathematics at Loyola Marymount University, **Alissa Crans** is known for her active mentoring and supporting of women, underrepresented students, and junior faculty. She shares her enthusiasm for math in settings ranging from school classrooms to public libraries to "Nerd Nite Los Angeles." Alissa continues to be intrigued by problems in the intersection of quantum algebra and geometric topology. Outside of mathematics, you can find her rehearsing with the Santa Monica College Wind Ensemble or on her quest to find the spiciest salsa in LA.

Glen Whitney trained as a logician, became a quantitative analyst at a hedge fund, and then founded the National Museum of Mathematics. From teaching at Harvard and Rutgers to leading public constructions such as an exhibit of polyhedra embodying Euler's formula, Glen continues to promote the importance of illustrating mathematics. He has a serial habit of editing problems columns: Varsity Math in the *Wall Street Journal*, The Playground in *Math Horizons,* and now the Prisoner's Dilemma for the *Prison Math Project*. He currently serves as a trustee of the Seattle Universal Math Museum.