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War Stories from Applied Math

Undergraduate Consultancy Projects

Robert Fraga, Editor

The Mathematical Association of America

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MAA

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Introduction

What good is mathematics? What is *supply-your-favorite-topic-commonly-taught-in-any-math-course* used for?

Have you ever had undergraduate students ask either of these questions? If not, you are in a very small minority of college math teachers, probably a set of measure zero. If you have had either of these questions asked, how do you answer? The first question is such a blockbuster that it invites a sweeping reply although one that could be given incrementally: Bar codes to bridges. The second question cannot be satisfactorily answered without some feel for the interests of the person asking the question. Suppose it is pre-med students who ask this question. They will be satisfied by an answer that relates to their professional interest. Geometric series, for example, have application in determining the concentration of a certain drug in a patient's blood system after a treatment of periodic injections, to cite an example drawn from an introductory course in calculus. But a business or an economics major will not find this answer particularly satisfying. Such a student will be happier with an example drawn from the accruing value of a bank account into which periodic payments are made.

This collection deals with these questions but from a somewhat different point of view. Problems are identified and, given the nature of those problems, the appropriate mathematics is brought to bear on their solutions. More specifically, the problems are drawn from sources in what is called–perhaps a tad pretentiously–the Real World. This is to say that mathematicians seek out problems from business, industrial, and local government sources and turn their students loose on these problems with varying amounts of assistance provided by faculty.

The origin for some of this collection was a workshop given at Marquette University in Milwaukee, Wisconsin, during the summer of 1996. This workshop, funded by a grant from the National Science Foundation (NSF), brought together four mathematicians involved, one way or another, in undergraduate consultancy projects, representatives from industry, and an audience of mathematicians interested in trying out the ideas presented by them. A unique aspect of the workshop was the inclusion of representatives of client companies who commented on their own experiences and their perceptions of student work in a business/industrial context.

The chapters that follow are adaptations of transcripts made at the workshop. We contend that the material herein is as relevant today as it was ten years ago, but with the passage of time, it has been necessary to update that material and each of the contributing authors has graciously done so. There are three supplementary chapters containing material that was not presented at the Marquette workshop: The opening chapter, *An Industrial Mathematics Program*, is based on a talk which George Corliss gave to an audience at the institution which is now called the University of Louisiana at Lafayette. It is an excellent overview of the subject of this collection. The two concluding chapters by Dan Maki and myself (Robert Fraga) are also supplementary. A sample student group report from the Math Clinic at Harvey Mudd College, in a slightly edited version, is provided as an appendix.

Let me say a few words about the principal contributors to this book.

George Corliss received his Ph.D. from Michigan State University. He has taught mathematics and computer science/engineering at Marquette University since 1978. A consultant with industry on a number of subjects, he has given talks to a variety of academic audiences on the practice and teaching of industrial mathematics.

Robert Borrelli received his doctorate from the University of California, Berkeley, and has been a professor at Harvey Mudd College since 1964. With industrial experience at Ford Aerospace and GTE Research Labs, he co-founded the Mathematics Clinic at Harvey Mudd and served as its director until his retirement.

Paul Campbell received a Ph.D. in mathematical logic from Cornell University. Working at Beloit College since 1977, where he served as Director of Academic Computing from 1987 to 1990, he has been Reviews Editor for *Mathematics Magazine* since 1977 and has been editor of *The UMAP Journal* since 1984.

Martha Siegel obtained her Ph.D. from the University of Rochester. From 1991 until 1996, she served as editor of *Mathematics Magazine*, one of many positions she has held with the Mathematical Association of America (MAA).

Daniel Maki has worked at Indiana University since 1979. His Ph.D. (1966) comes from the University of Michigan. In 2004, he was a recipient of the President's Award for Teaching Excellence.

Tom Davis was Senior Vice President of Super Steel Products Corp at the time he contributed his chapter to this book. Previously he had served as Dean of Faculty of the Milwaukee School of Engineering. His consultancy roster included more than four hundred companies and five hundred projects over a wide variety of topics.

Robert Fraga got his Ph.D. from the University of British Columbia in 1965. Currently at Baker University, he has worked in the calculus reform movement and on undergraduate consultancy projects, the subject of this book.

The authors who have contributed to this book come from a variety of backgrounds, and their remarks, although focusing on different aspects of consultancy projects, indicate how such projects can be carried out in a variety of academic contexts. Represented in this book are

- A large Jesuit university in an urban setting
- A small, prestigious college focusing on mathematics, science and engineering
- A state-supported university serving both residential and commuter students
- Two liberal arts colleges, one in a small city, the other in a rural environment
- A multi-campus state university

The workshop at Marquette concluded, not surprisingly, with the question "Where do we go from here?" Participants were inspired to emulate the practices described by the principal speakers, and some of them, in fact, returned to their home institutions where they conducted successful consultancy-type projects. But the ideas expounded at the workshop beg a wider audience, and that has provided the impetus for the creation of this book. It aims to promote an activity that has proven beneficial to its authors. The structure of the book is provided by various aspects of consultancy work, and these are generally suggested by the chapter titles. Running in parallel with this structure is an underlying theme: The variety of contributors and the institutions at which they work indicate that student consultancy work can be successfully undertaken over a wide spectrum of academic institutions. The hope is that this will provide the encouragement necessary to motivate readers to start programs of their own.

Acknowledgements

I owe a debt of gratitude to Donna Marquart of Ripon College who spent many hours transcribing a set of tape recordings from which much of this book has emerged. Thanks are due to the National Science Foundation (NSF) under whose auspices was held the workshop alluded to above and to Marquette University in Milwaukee, Wisconsin, which hosted it. Julia Gonzalez of Baker University learned LaTeX in order to retype the student report given in the Appendix. My colleague, Denis Popel, also of Baker University, provided invaluable assistance in preparing the book for publication and in formatting it.

Lastly I need to thank the reviewers of the book for providing many insightful comments, some of which I have shamelessly cribbed in revising the text. Mistakes and lapses of judgement, however, are entirely my own and no one else's.

Robert Fraga
April 2006

An Industrial Mathematics Program

George Corliss[1]
Marquette University

Introduction

Our program at Marquette is a small do-it-yourself operation, but you might still get some useful ideas from it. This chapter tells the story of what seems to work for us, with all its warts and struggles. I offer these observations not as an authority but as one who has had some experiences others might use.

Industrial Mathematics in the Marquette University Department of Mathematics, Statistics and Computer Science (MSCS) has the following components:

- Faculty orientation and perspective
- Influence of biological mathematics
- Course projects
- Course offerings
- Work experience
- Industrial-based graduate research assistants
- Career counselling
- Consulting and research

It is a high faculty involvement, low University and industry cost program that produces well-educated, marketable mathematicians at both the graduate and the undergraduate levels. By "high faculty involvement," I mean that the faculty devotes significant effort to it.

This chapter reports on a program in industrial mathematics that has evolved over a period of more than 20 years and has involved many people. Much of the leadership and cultivation of the climate has come from our Department Chair, Doug Harris. Other faculty who regularly teach in the mode described here include Steve Merrill, Gary Krenz, Peter Tonellato, Anne Clough, Naveen Bansal, and myself, George Corliss.

What IS industrial mathematics?

"Whatever we define it to be, nothing more, and nothing less." — after Lewis Carroll.

Not: "If the only tool you have is a hammer, the whole world looks like a nail." — Mark Twain.

[1]Based on a talk presented at University of Southwestern Louisiana, October 24, 1996. The work has been supported in part by several grants from the National Science Foundation and in various ways by many industrial partners.

Anything we do that is useful in industry — Marquette Department of Mathematics, Statistics, and Computer Science.

If we promise to teach "Industrial Mathematics," we ought to be able to define "Industrial Mathematics." I would rather not, for fear of leaving out something I might like to do. We try hard not to define ourselves into too small a box. We teach mathematics, statistics, and computer science, and we try to use those as tools in the advancement of science and engineering. We try hard not to care whether the work is called mathematics, computer science, biology, medicine, or engineering. We try hard to remain open to any application we find personally interesting or which uses tools we know or can learn.

I do interval arithmetic, and I know about differential equations which are not very stiff. Suppose I advertise, "Hey, I'm your man if you want validated bounds to differential equations that are not stiff." I don't think I'll get much interest. But if I say, "Let me come and talk about things of concern to you; there may be a chance that I can discover something or that I know something or that one of my colleagues knows something that might be helpful to you," then that will open doors. That's why I don't define industrial mathematics too narrowly.

Who are "we" mathematicians?

The faculty in our department were all trained as mathematicians, and we think of ourselves as mathematicians. What do we have to offer to industry?

- General-purpose problem solving
- Precise use of language
- Specific tools from mathematics, statistics, and computer science
- Asking questions
- Promising only what we can deliver

Mathematicians are trained to be general-purpose problem solvers. We divide and conquer, taking big, hard problems and breaking them down into smaller and smaller pieces which we attack systematically, bringing to bear on them whatever it is that we know.

We're trained to use language precisely. There are whole fields of mathematics in which people study the consequences which follow from the precise definition of just a few words. Precise use of language is so essential in so many kinds of business applications that we might assume that everybody is systematic about things. They're not. This is a unique skill that mathematicians possess. We also have specific tools — mathematical, statistical, and computational. We're good at asking questions.

Our work has both positive and negative consequences, but we're a modest lot. We don't brag much. In a positive sense, that means that if a mathematician tells you something is going to happen, the chances are pretty good that it will happen.

What is the Marquette MSCS?

To help you see where Industrial Mathematics fits, let me tell you a little about Marquette and our department. Marquette is an urban, national, Jesuit university with about 10,500 students in 50 undergraduate majors and 35 graduate programs. We offer strong programs in liberal arts (our college), engineering, and business, but our primary strength is liberal arts undergraduate education.

We are a Department of Mathematics, Statistics, and Computer Science (MSCS). We try very hard not to draw boundaries between those disciplines. One faculty member who teaches computer architecture has a Ph.D. in Classics. The research interests of faculty teaching databases and compiler construction are abstract algebra, logic, and foundations. This kind of cross-disciplinary culture enriches what we have to offer, and our students benefit as a result. I remember a student who was excited one semester to find that his professor for Greek was the same person who taught Pascal. That is a strong model for a liberal arts education.

Our faculty of 24 offer service courses for Engineering, Business, and other Arts and Sciences students. We have excellent working relationships with many other departments. We graduate about 30 undergraduate majors and about 20 Master's (MS) students each year. There is little undergraduate/graduate separation. Since graduate students can take several advanced undergraduate classes, sometimes more than half the students in an "undergraduate" class are graduate students.

What constitutes an undergraduate degree?

All students in the College of Arts and Sciences complete a core curriculum of about 60 credits (of 128 credits required for graduation) in English, history, foreign language, theology, philosophy, and the social and natural sciences. Students studying industrial mathematics may major in mathematics, computer science, or computational mathematics.

Mathematics major: Requires 13 courses, including calculus (3 courses), discrete mathematics, linear algebra, "pure" math (3), computing (1), and four additional courses (4).

Computer Science major: Requires 15 courses, including calculus (2 courses), discrete mathematics, computational models, and 11 general computer science courses.

Computational mathematics major: Requires 16 courses, representing a combination of mathematics and computer science.

Students interested in statistics usually complete a mathematics major.

What constitutes a graduate degree?

Graduate students receive a degree from the Department of Mathematics, Statistics, and Computer Science. They may claim certain emphases, but there is no official specialization at the MS level. Many of our graduate students take as much computing as they can, but we are careful to say clearly that we do not offer a MS in computer science because we have no faculty doing research in computer science. In practice, we prepare excellent systems analysts, which we include under the "industrial and applied math" umbrella.

A Master's degree requires 10 courses, 6 at the graduate level, including one-year graduate sequences in two areas of mathematics. We offer a Ph.D. in the areas of biological mathematics and in algebra, two areas in which we have significant faculty research programs.

Do you have an Industrial Mathematics Program?

If I am talking to university administrators, accountants, or attorneys, I say, "No." If I am talking to students, colleagues, or employers, I say, "Yes."

No? We have nothing so designated by our department or our university. We have no major. We have no budget. We have no designated graduate degree. We have no Industrial Mathematics faculty. We have no set of designated course offerings.

Yes? We do an excellent job of preparing students to use their math skills for jobs in industry. We have many faculty with active industrial contacts. Our personal industrial experiences illumine our teaching. We have about 15 courses with varying degrees of industrial content. We have excellent computer laboratory facilities.

By claiming no "program," we are under no contractual obligations. If I decide to offer a more standard course, or if I leave, the department is under no obligation to continue to offer classes in the style I have been doing. We have succeeded in keeping a low profile, promising little and delivering significantly more than we promise.

What are the components of the Industrial Mathematics Program?

The program has several components.

Faculty orientation and perspective: Individual faculty approach their teaching assignments with the intent to teach useful material. Individual instructors have a high degree of autonomy in the classroom.

Biological mathematics: An active research and instructional program in biological mathematics complements effectively the Industrial Mathematics Program.

Course projects: Most of the courses we consider to represent industrial mathematics use projects to varying degrees.

Course offerings: We offer about 15 courses in industrial mathematics. These same courses are also in several other programs. Courses considered in the program are characterized more by the perspective of the instructor than by any formal designation.

Work experience: We encourage students to participate in some real work experience. All undergraduates wanting such experience are placed. The placement of foreign graduate students is complicated by US immigration policies, but many of them also receive practical work experience.

Industrial-based graduate research assistants: NSF and industrial partners have jointly funded several industrial-based graduate research assistants. Students work half-time at industrial sites.

Career counselling: The department's Work Experience Coordinator conducts an active, informal program of counselling students. We work closely with our Career Center for job placement.

Consulting and research: Several faculty engage in independent industrial consulting and industry-sponsored research. Those industrial experiences enter into our classrooms every day.

How did we get here?

Our program is highly flexible and loosely structured. It has evolved into its present form gradually over at least 20 years, and it continues to evolve. Somehow things changed and people started doing things a little bit differently. There was no time when anybody said, "The world as you know it is now over. We're going to do things differently in the future." Instead, several of us said, implicitly or explicitly, "It would be better if we included more useful stuff in some of our classes." We evolved in that direction, following the interests of several faculty.

One way to make our classes a bit more useful is to use projects in them. That is a key component in the way I teach. In order to do that, it is important that instructors have a considerable amount of autonomy in their classes. Of course, our undergraduate courses feed into our graduate level classes, so there are certain topics that students ought to know when they leave those undergraduate classes. However, it is my call whether I want to do things in Fortran, or in MATLAB, or if students are sent out to work for a company.

We tend not to have very many faculty meetings, but we do a lot of informal talking in the hallways. Most of the coordination and consensus-building happens *en route* to the coffee pot.

How is Industrial Mathematics related to Biological Mathematics?

We have a strong biological mathematics component. Every week five of my colleagues spend time in a laboratory at the Medical College of Wisconsin, at the Zablocki Veterans Administration Medical Center (VA), in our Department of Biomedical Engineering, or in our Department of Biology. Our graduate students put on lab coats and do some of the laboratory work that generates the data that they analyze. I find that exciting. So do they.

Earlier, I said we try hard not to define ourselves into too small a box. I do not think there is any significant difference between industrial and biological mathematics, although some of my colleagues would disagree. We do not have a formal program in either one, although we *do* have a Center for Biomedical Engineering and Biomathematics.

Biology, medicine, biomedical engineering, and other concerns of the biological mathematician fall well within our definition of industrial mathematics. Hence,

$$\text{Biological Mathematics} \subseteq \text{Industrial Mathematics}\ .$$

On the other hand, a biological mathematician may need all of the tools covered in industrial mathematics classes. Hence,

$$\text{Biological Mathematics} \supseteq \text{Industrial Mathematics}\ .$$

We view the two programs as complementary. Whatever differences there are lie in the relative interests and emphases of particular students, faculty, or employers, and in the content knowledge necessary for a particular application.

What is a Work Experience Coordinator?

Some of the glue holding the Industrial Mathematics Program together is provided by the Departmental Work Experience Coordinator, which is part of departmental committee responsibilities. I created the position whose job description follows:

- Keep things informal
- Act as a clearinghouse for job openings
- Recruit students for jobs
- Visit employers
- Visit currently working students
- Visit and correspond with former students
- Do some career counselling
- Talk to student groups and classes

I disseminate job and career information by

- “In-the-halls” advice
- Informal advising
- Formal advising
- E-mail, Web page
- Physical postings
- Faculty meetings
- Admissions, Career Center, Business, Engineering connections

Year-round, my telephone log averages more than one call or e-mail message per day from potential employers.

What classes in industrial mathematics do you offer?

Classes typically taught in the spirit described here include:

- Calculus
- Computational Models (since 1975)
- Biological Mathematics (since 1982)
- Differential Equations
- Boundary Value Problems
- Statistics
- Numerical Analysis
- Operations Research — constrained, stochastic (since 1982)
- Computer Systems Analysis (since 1980)
- Computer Networks (since 1982)
- Industrial Mathematics
- Special Topics courses.

How are our classes taught?

Our instruction features

1. concepts
2. practice
3. projects

We make heavy use of "war stories" as instruction. We often draw outside speakers to share their "war stories." In one sense, this chapter is the war story about our program. I cannot tell you what will work for you, but I *can* tell you the story of what seems to work for me.

War stories?

I tell many war stories. That is what I'd like to do now. Instead of dwelling on theoretical issues, I like to say, "I remember the time when . . .." We all remember when our grandparents used to talk like that. We'd roll our eyes and say, "There goes Grandpa again," but we remember those stories.

Once in my operating systems class, I wanted to talk about scheduling processes and how you keep track of different processes. I started to go through the book's discussion. Everyone was falling asleep. The class runs from 5:45 to 7:00 p.m., and a third of the students work full-time. They'd already put in an eight, nine, or ten hour day. You can imagine the result.

"I remember the time when I was working on a project with Delco Electronics. We were trying to keep track of an aircraft inertial guidance system. We had the monitors, gyroscopes, and accelerometers."

Since I had to integrate the differential equations of motion, I got to sneak a little bit of numerical analysis into a computer operating systems class while I talked about process management. I smiled. The students frowned. Without that kind of approach, most of these students, some of whom are as old as I, would have fallen asleep. Now they sit up and pay attention. You need to talk about concepts, but sometimes the way to get those concepts across is to use war stories.

I make heavy use of outside speakers in my class. Honestly, I don't think that many of the people from outside do as good a job as I do, but they get paid a lot for doing their jobs, and the students respect that, so they often are more effective than I.

Whom are we trying to please?

Our relative priorities are as follows.

1. **Ourselves:** We do what we do in the program because we think it is the best way we can do our jobs and because we derive intense personal satisfaction from doing it this way. To us, the motivation is very similar to the motivation for studying pure mathematics, "It is beautiful, and it pleases me to do it well." Part of the beauty of industrial mathematics is that it is also useful. Do not underestimate yourselves as an audience to be served.
2. **Students:** We do our best to attract, educate, and graduate high quality, motivated students. Much of what we do has proven, in our subjective judgments, to be highly effective in helping students learn material that is interesting and useful.
3. **Industries:** We prepare students for industrial jobs. We try to provide skills employers need, and we work closely with employers so that we know what their needs will be. However, we are more interested in the education and long-term preparation of students than in satisfying short-term industrial needs, either for job skills in new hires or in projects we undertake. That means that we often decline industry-sponsored projects that do not fit our instructional goals. We are an educational institution, not a consulting company.
4. **Funding sources:** If we can get support for providing quality instruction, we are eager to do so and can be quite flexible. However, we do not chase funding.

Where do our clients come from?

Our projects come from a variety of sources. These include:

- Students finding their own; this builds ownership and confidence.
- Previous successful projects.
- Word of mouth.
- Former students; most of our former students are working, and we talk to them.
- Informal and social contacts.
- Career Services.
- Alumni.
- Research Support.
- Consulting.

Most of our students find their own projects. In my systems analysis class, for example, the students walk in the first day, I form them into groups of about four or five, and I give them their first assignment: They have three weeks to find a client and write a proposal for the systems analysis work that they're going to do for that client. Their assignment for the rest of the semester is to do whatever the proposal says. What my class does is up to the students, since they propose the work they will do for their clients. My job is to provide them with the information required to do that.

Some students are horrified about what I'm making them do. Isn't the professor supposed to give you the course syllabus? What's all this about being sent out to find a project to work on? After about a week, almost every group has a pretty violent argument about which of the four or five projects they found they really want to do. They don't have a hard time finding the projects. If they find their own project, it's their project, not mine. When they screw up, whose fault is it? The students have a lot more invested in their own project. They are working for somebody they know, so they really want to do a good job for this client. If I had given them the client, that wouldn't be the case.

It is important to ask for clients. In 1984, the first time that Jesse Jackson was running for President, I had an opportunity to attend a rally. Although I don't remember any of the political statements he made, I *do* remember very clearly that at the end of his talk, he gave an altar call for money, "If you have any money, come on down and write a check." I also remember very clearly that he said, "Some people object. Some people think it's un-presidential to ask for money. I'll tell you what's un-presidential: it's not having any. So if you've got some, bring it on down." Some people think it's un-academic to ask for students or to ask for money. I'll tell you what's un-academic: it's not having any money or any students. Don't be afraid to ask. Get over any reservations about embarrassing yourself. It's much more embarrassing to get fired because you don't have any students.

What to do? LISTEN

Recall "If the only tool you have is a hammer, the whole world looks like a nail." — Mark Twain

Once we get to see a potential client, what do we do? We don't say, "I'm an expert in interval solutions to differential equations." We listen. We say, "Tell me about your concerns. Tell me about what you do." We ask a lot of questions. I've talked to lots of companies over the years, and I've developed a pretty good idea about what questions to ask. For example, I've sometimes seen problems similar to the one under discussion, and I say, "Well, are you having trouble like this?"

The problem is *never* what either the client or I first think it is. This may be the single most important lesson the student (and often the client) learns from the project.

Projects are driven by the science or the business, not by tools. This is one of the characteristics of good industrial mathematics or good systems analysis. I try to steer a project toward the intended content of the course, but I am prepared to help the students do whatever is necessary to address the client's problem. We try hard not to define ourselves inside too small a box.

Who does what?

Whatever it takes. Typically,

- **Clients** provide access, information, feed-back. High levels of client commitment are essential.
- **Students** ask questions, provide legwork and analysis.
- **Instructor** provides guidance.

What kinds of projects?

The most successful projects

- Involve working with scientists and engineers, not vice presidents.
- Interest our scientific contact who genuinely owns the project.
- Advance the business goals of the client.
- Prove challenging; it is far easier to restrict scope than to grow a toy project.
- Are long-range; students have the time to spend days in background research.

There are several advantages to projects on this scale.

Students recruit: Less preparation is required of the instructor.

Easy approval: Sponsoring scientist usually has the power to say, "Yes."

Limited risk (for client and for me)**:** Work of student teams cannot be guaranteed, although we have a better record than many professional consultants.

Many available: If the client does not need to pay significantly, students find that there are MANY projects from which to choose.

Less legal entanglement: When negotiating for projects of about three months duration, it helps to get started quickly; the sponsoring scientist, the students, and I can shake hands and say, “Let’s do it!”

Some factors seem to contribute to unsuccessful projects.

- Client does not care, is unavailable, or was forced.
- Client is doing students a “favor.”
- Client needs slave labor.
- The project is too small, routine, or well-defined. There is no opportunity for analysis.
- It is not critical to the business.
- No access to content knowledge is provided.
- Most information is confidential or proprietary.

The worst projects my students do are those which are finished about the middle of the semester, leaving the students with nothing to do. The best projects are the really hard, wide-open ones which the client company wouldn’t dare submit to a consulting group because the client couldn’t write down the job description of exactly what was wanted, and the consulting company wouldn’t know how to price the project. It’s no problem to turn students loose on it. They can go off and club away at the problem. If they succeed, Great! If they don’t succeed, well, the company didn’t lose very much.

I am more successful if I work with the scientists and the engineers who have the problems rather than with the vice presidents who have the money. That’s one of the main points that distinguishes what I do from what the people at Harvey Mudd College (HMC) do[2]. HMC has somebody whose full-time job it is to schmooze vice presidents. They charge their companies $40,000 to do a project, so they’ve got to find someone with the authority to sign a check for such a sum. There is a risk that the students who work on the project are not going to produce anything. That’s a risk I don’t have to take. I go directly to the scientists and the engineers who have got a problem. They usually have the authority to say, “Yeah, we can work together.” Then I have somebody with a vested interest in getting something done on the project. It’s important that the clients feel that they own the project and have a genuine interest in it. Some of the worst projects that I’ve done have been for groups inside the university who felt they were doing me a favor to work with students. They were condescending and treated the students like children. I want a client that really wants to make sure that the work gets done. I want to be doing the client a favor, not the other way around.

What does the client pay?

For a while, I didn’t have the guts to ask for money. Then I heard the speech by Jesse Jackson. I still don’t have the guts to ask for much money. I’m not a gambler. Although I have a track record for this sort of work, the students doing the projects don’t, and I have reservations about asking for very much money for an unproven product. As a compromise, I expect the client to pay the students’ out-of-pocket costs. I began this policy after one client insisted that the project team provide them with 25 copies of each project deliverable. That’s right: it was a governmental client.

Clients receive work of value, often nearly 500 hours of work. That work usually has *much* more value to them if they pay even an insignificant amount. I encourage clients to evaluate the completed project and to pay the students 10–20% of the value they assign to the project. Several clients have paid the students $100 each. We do not charge the client anywhere near the value of the work we provide.

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[2]See the following chapter

- It is *much* easier to recruit clients, especially for the students to recruit their own clients, if there is no financial cost.
- We can conduct negotiations with a project's scientific sponsors, rather than with vice presidents, accountants, and attorneys.
- The risk to the client, the students, and me of a possible failure is minimized.
- The client has little leverage to demand anything. This reduces stress on the students (and on me).
- Student teams have insufficient experience at the beginning to draft the contract, project schedule, and deliverables that would be required for a formal contract.
- No cost leaves clients with modest expectations, which we can usually exceed. To have a satisfied client at the end of the semester is a great reward for the long hours the students invest.

I am currently considering a client who is volunteering to pay for a project. I will probably choose *not* to take on the project on that basis. I am concerned about the class dynamics if one team is being paid and the others are not. I also think it is unjust for the university or the department to make money from students' labor.

What sort of projects do you do?

Here is a sampling of course projects from the past few years.

Mathematical Modeling:

Miller Brewing
Purpose: Adjust the pressure difference in the Lautering tun to eliminate the need for back flushing.
Source: A student made a cold call to the number in the telephone directory and said, "I'd like to do a mathematical modeling project." It took him something like twelve relays before he got to somebody who said, "That sounds interesting. Why don't you come over and talk to me?"

He ended up optimizing a little piece of the brewing process. At their plant on the east coast, Miller had just replaced seventeen of these tanks at a cost of a million dollars apiece. Because of the optimization that he did on the pressure control of each tank, the company didn't have to replace the tanks at their plant in Milwaukee. At the end of that project, the scientist my student worked for wrote him a letter that said, "You saved Miller $17,000,000. Any time you'd like to come and work for us, you've got a job."

This student is now a high school math teacher. His students know something about mathematics, and they know what it means to apply mathematics.

Numerical Analysis:

Milwaukee Sewerage District
Purpose: How do we adjust the flow rates in segments of the system under various demands? Network flow optimization.
Source: Civil engineering student's advisor had consulting contract.

Wisconsin Gas Company
Purpose: Reliable machine recognition of symbols from scanned blueprints.
Source: Student had part-time job.

Medical College of Wisconsin and local medical instrument company
Purpose: Can chaos in heart rates be used for diagnosis?
Source: Another faculty member is on the research team.

Medical College of Wisconsin
Purpose: What information can we extract from wrist movement sensors?
Source: Student engineering advisor is on the research team.

Nonlinear Optimization:

Sun Microsystems
Purpose: Software for global optimization.
Source: Sun's project manager is a long-term research colleague.

Genome Therapeutics
Purpose: Gene prediction.
Source: Another faculty member is on the research team.

MacNeal Schwindler
Purpose: Design rocket exhaust nozzle.
Source: MacNeal Schwindler contacted Sun.

BancOne
Purpose: Portfolio management.
Source: Recruited by a faculty colleague in the Department of Economics.

Swiss National Bank
Purpose: Currency trading.
Source: Recruited by a faculty colleague in the Department of Economics.

GE Medical Systems
Purpose: NMR antenna design.
Source: Recruited by a faculty colleague in Electrical and Computer Engineering.

GE Medical Systems
Purpose: AC motor design.
Source: Recruited by a faculty colleague in Electrical and Computer Engineering.

Computer Systems Analysis:

Nativity Jesuit Middle School
Purpose: Computer laboratory for instruction.
Source: Two team members were volunteer teachers' aides.

MagnaStar
Purpose: Bug-tracking system.
Source: Client is a former student; team member has a part-time job.

Publishing Perfection
Purpose: On-line auction.
Source: Client contacted me to hire a student intern.

Eaton (4 projects)
Purpose: Systems analysis of Eaton's systems analysis group.
Source: Student had co-op job with a neighboring group.

Purpose: When will an injection mold need replacing?
Source: Continuation of a student's co-op assignment.

Purpose: Expert system for selecting a vendor for injection plastics mold.
Source: They liked us the previous year.

Purpose: Web site to support software component development.
Source: Client contacted Office of Research Support, then discovered they liked us the previous year.

Wisconsin Funding Information Center
Purpose: Develop an easy-to-use commercial product to access information about foundations in Wisconsin.
Source: Client contacted me.

Quad Graphics (printing, 4 projects)
Purpose: Prototype Web technology as a front-end for subscriber data base.
Source: Student contacted friend, a former student, who works there.

Purpose: Manpower scheduling on a printing line.
Source: Piggy-backed on previous contact.

Purpose: Prototype Web technology as a front-end for subscriber data base.
Source: Continuation/extension of previous year's project.

Purpose: Design customer fulfillment system.
Source: Continuation/extension of previous year's project.

J.J. Keller (OSHA training, record-keeping, compliance)
Purpose: Develop a corporate Web site.
Source: Referral by student in another class whose father owns the company.

City of Oak Creek
Purpose: Develop a city-wide information plan.
Source: City Engineer contacted the department.

Carpenter Brothers (foundry sand)
Purpose: Computerize office functions, order entry, sales information, inventory.
Source: Team member knew an employee.

Milwaukee Ballet
Purpose: Organize subscriber and donor records.
Source: Team member interested in ballet did volunteer work there.

GE Medical Systems
Purpose: What technologies should we be considering for the next generation of global communications networks?
Source: Supervisor of former student contacted me.

Johnson Controls
Purpose: Develop an internal testing and evaluation plan for modems and for PCMCIA cards.
Source: Team member had a part-time job there.

MU Bookstore
Purpose: Write a "Request For Proposal" for a new ordering, cash register, inventory control system.
Source: Another class member had a part-time job there.

MU Library
Purpose: Analyze information flow in preparation for computerization of card catalogue.
Source: Team member worked there.

MU Equal Opportunity Program(EOP)
Purpose: Computerize office record-keeping, appointments, contacts.
Source: Team member participated in EOP.

Web Site Design:

Class project teams have developed Web sites for:

Milwaukee Youth Symphony Orchestra
Source: Mother of a team member is the Executive Director.

JQuiz (provide quiz about current events)
Source: Client is a former student in journalism.

Wisconsin Mental Health, Substance Abuse, and Aging Coalition
Source: Client is a former student.

Catholic Herald (newspaper)
Source: Faculty colleague in the Department of Journalism is on the Board of Directors.

MU College of Arts and Sciences
Source: Continuation of a previous year's project.

MU Career Center — MU SCAN
Source: I work closely with the Career Center Director.

MU College of Engineering
Source: The Dean was impressed by the previous year's projects and contacted me to hire a student intern.

MU Memorial Library Archives (2 sites)
Source: Recommended by the previous year's client.

Jade Technologies
Source: Client contacted me to hire a student intern.

Wisconsin Geriatric Education Center
Source: Employee registered for the class.

MU Haggerty Art Museum
Source: Director contacted me.

MCW Bone Marrow Transplant Registry
Source: Employee registered for the class.

Pflow Industries
Source: Employee registered for the class.

MU Department of English
Source: Graduate student registered for the class.

Independent Study:

OnContact (contact management software)
Purpose: Remote database maintenance and synchronization.
Source: Employee enrolled in the course.

Delco Electronics
Purpose: Aircraft inertial navigation system programmed in Ada.
Source: Manager contacted me for Ada training.

Repete (industrial process controls)
Purpose: Object-oriented design of an asphalt plant in C++.
Source: President is a former sailing buddy.

How do you attract new students?

We wish you good luck! Our luck has not been outstanding. It is important to know ourselves and understand what we do. We continually remind ourselves that there are more C students than A students, and that the C students pay the same tuition. We tell the story to the following groups:

- each other
- present and prospective students
- alumni
- the administration
- the Career Center
- Admissions

What work experiences are available to your students?

A variety of available work experiences includes:

- Volunteer
- Part-time
- Summer
- Class project
- Independent study
- Co-op
- Full-time
- Faculty research project
- Graduate research assistantship

Our Work Experience Program is extremely flexible in order to meet the needs of students and employers, under the direction of the Work Experience Coordinator. That means that we will talk to students and employers about any work arrangement that makes sense. We do not restrict co-ops to four alternating work-study periods, and we do not specify a minimum or a maximum number of hours for a part-time internship.

Do you give credit for co-ops and internships?

For co-ops (work full-time for a semester and return to school), we give one credit, primarily as a device for retaining the student in full-time student status to retain student loan deferments, continued inclusion in parents' insurance, and eligibility for basketball tickets.

For internships, no. Faculty sentiment is divided. Work experience such as an internship is extremely valuable, both in what the students learn and in improving their employment prospects. Awarding credit would encourage more students to take advantage of the opportunity; however, just because work experience is good does not mean it should have the reward of academic credit added to the existing academic, creative, career, and financial rewards. Students' records for employment may be *stronger* if they do *not* receive credit. They can show the internship whether they receive credit or not. If they do not receive credit for the internship, they have an additional course on their transcript, and the total cost is the same. Furthermore, it is not fair for us to accept tuition payment for the relatively small value we add to an internship.

What are the sources of work experiences?

Generally the same as for class projects. Opportunities are plentiful right now; we have several times more openings than we have students to fill them.

How do you place students?

We are very successful. Our students are in very high demand. How do we do it?

They do it: Most students find good jobs without any explicit help from us.

Have a history: Nothing succeeds like success. Most students are hired by firms that have hired our students before.

Career Services Center: We work closely with the Career Services Center so that our students and faculty know its services and so that the Center knows what skills our students offer.

Networking: Talk about job openings and skills of our students at every opportunity, formal or informal, business or social. We regularly invite firms to provide speakers for classes or student groups.

Faculty attitude: Faculty values employment as an outcome of university education.

Recruit interviewers: We invite firms to visit campus, and we talk with interviewers while they are here.

E-mail and Web: We use technology to communicate with employers, with potential, current, or former students, and with other contacts.

Former students: We cultivate them and encourage them to remain in touch. We define "alumni" as "anyone who is willing to be associated with us." We count many engineering or business majors among our "alumni," and they are valuable contacts.

Computer accounts: We offer alumni computer accounts.

Departmental newsletter: We publish (physically and electronically) a semi-annual newsletter featuring many letters from former students and current events in the department.

Christmas cards: I exchange Christmas cards each year with more former students than my wife would wish.

What is the role of consulting by faculty?

Several colleagues and I do some independent consulting. We sometimes involve students in those projects. We often discuss these projects in our classes.

I have developed programming courses in Ada and in C++ for companies and then offered similar courses for our own students. My consulting experiences walk into my classes every day.

Biological mathematics research plays a similar role for four other colleagues, including joint research with the Medical College of Wisconsin, the Zablocki Veterans Administration Medical Center, Marquette's Dental School, the Departments of Biomedical Engineering, Electrical and Computer Engineering, and Biology.

We're no snobs; we'll talk with anyone. Our buildings have walls to keep Wisconsin winters out, not to impede the flow of ideas.

What sort of industrial consulting do you do yourself?

Here are my own major industrial consulting projects:

Amoco Production
Interval methods, numerical solution of partial differential equations for portions of an oil reservoir simulation model. Contributed to the eventual decision of an Amoco employee to attend Marquette.

International Mathematical Subroutines Library (IMSL)
Numerical and statistical analysis, Ada software development.

Numerical Algorithms Group
Interval analysis, Ada software development.

IBM
Interval analysis, validated computation. Developed software package for self-validating adaptive quadrature. IBM marketed our package as part of the ACRITH product.

Argonne National Lab
Automatic differentiation of algorithms, numerical analysis, optimization, software development. Team developed software currently being used by the Department of Energy, Boeing, General Motors, AT&T, NASA, and many others.

Cardiac Evaluation Center
Numerical analysis, digital signal compression, analysis, and display for medical data. Resulted in a marketed product.

Delco Electronics
Modeling motion, numerical analysis, Ada software development for commercial aircraft positioning instruments.

Strong Mutual Funds
Financial modeling, systems analysis.

North Sails
Numerical analysis, computer graphics, software and user interface design for the design and manufacture of yacht sails. This technology has appeared on every America's Cup contender, both US and foreign, since 1980.

US Olympic Team
Numerical analysis and computer graphics for the design of bobsled runners. In 1983, long before the winter 1994 excitement about the Bodyne sleds.

Harken
User interface, database design, software development for marketing yachting equipment. Harken is the world's leading supplier of high-performance yacht racing equipment.

Prent
Mathematical modeling, linear programming, statistical analysis of optimum box size for shipping manufactured parts.

BTI, Inc.
Pattern recognition, statistical analysis of light reflectances in satellite or other remotely sensed data.

Repete Corp.
Systems analysis, computer graphical display, software development for industrial process control.

NISA
Selection of software for large-scale numerical optimization.

What are your faculty rewards?

My cynical answer is, "In your **dreams**!" We receive no release time and no overload pay for the activities I have described.

My personal answer is that we receive tremendous satisfaction in a job well done, especially in seeing the career trajectories of ordinary students. Some more practical issues include:

Part of the job. The activities described here are part of our normal jobs. I have decided that this is the most effective way to teach the classes I am assigned to teach.

Authoritative teaching. By practicing industrial mathematics ourselves, and by allowing our students to experience industrial mathematics in their projects, we bring authority to our teaching. "The people were amazed at his teaching, because he taught as one who had authority, not as the teachers of the law." Mark 1:22.

Thanks for doing your job? Should I expect rewards for doing my job? Isn't this what is required of me?

Teaching IS recognized. At Marquette, teaching is recognized as a factor in promotion, tenure, and salary decisions.

Consulting as research? Consulting rarely results in refereed journal publications, but it does promote many of the virtues usually cited for research: intellectual vitality, creation of new knowledge, impact on classroom instruction.

External funding. Opportunities for external funding are *much* greater in industrial and applied mathematics than in pure mathematics.

Personal consulting. The project-based instructional style I have outlined leads naturally to faculty consulting opportunities.

What resources and funding are required?

There is one critical requirement: committed, dedicated faculty. Not much is required that individual instructors cannot provide. The department needs to provide understanding, encouragement, and "in the halls" coordination. The university needs to provide computing equipment, library facilities, and an openness to scientific research, not just mathematics. Industrial partners need to provide the time necessary to work with student project teams.

At Marquette,

- NSF has funded a $100K computer lab.
- Sun, NSF, and Abbott Laboratories have funded a $250K JavaStation classroom.
- NSF and industrial partners have funded 4 GRA's.

What is your program evaluation?

My cynical answer is, "Yeah. Right." In practice, we have nothing formal, not even good student tracking. However, we do have *many* post-graduation contacts and many letters, messages, and phone calls from enthusiastic former students. We also have employers waiting at the doors to hire more students like the ones they hired last year.

How can I do something similar at my place?

In pseudocode:

1. It is OK to start small and evolve.
2. Consider a Work Experience Coordinator.
3. For individual faculty members:
   - Decide
   - Use projects in current courses
   - Cultivate industrial contacts, especially with former students
   - Talk to each other
4. In time
   - Evolve course content
   - Evolve course offerings

Epilog

Since the talk on which this chapter rests was delivered in October, 1996, much has changed, and much remains the same. I have moved from the Department of Mathematics, Statistics and Computer Science to the Department of Electrical and Computer Engineering, hoping to find the climate in engineering a bit more encouraging for applications and for students' careers. If anything, my reliance on project-based instruction has increased. For example, in the Fall 2003 term, I tried for my first time having the entire class work together on a single project, the design and implementation of a web search engine.

The Department of Mathematics, Statistics and Computer Science has added faculty members in bioinformatics, applied statistics, and applied computer science. Their teaching fits into the "industrial mathematics" framework, and one of the new faculty has assumed the Work Experience Coordinator role I left. Students' employment prospects are not as rosy as they were in 1996, but if anything, Marquette is stronger now in preparing industrial mathematicians than it was in 1996.

References

[1] Avner Friedman and John Lavery, *How to Start an Industrial Mathematics Program in the University,* SIAM, 1993.

George Corliss works at a Jesuit university in an urban environment. He currently is a professor in the Department of Electrical and Computer Engineering although at the time he contributed to this book, he worked in the Department of Mathematics, Statistics, and Computer Science.

Source of Problems: Industrial Contacts

Robert Borrelli
Harvey Mudd College

Harvey Mudd College (HMC) students may be involved in an industrial mathematics project through the college's Mathematics Clinic which runs for a full academic year. We call our program the Mathematics Clinic, a name which exposes us to some criticism. In mathematics, a "clinic" is a place for remedial work, right? You go to the clinic if you can't do the problem. At HMC, the word "clinic" is more closely related to what hospitals do. If you are sick, you might go to a major university hospital that has a board of experts, and this board of experts will analyze your problem and solve it for you. That's where the clinic idea came from. We didn't invent it; we took it over from the engineers who named what they set up in 1964 the "Engineering Clinic."

We mathematicians were invited to become involved in their program, but we never really were partners. We could be consultants and spend time on projects, but we never got release time to do anything, nor did we get any credit for what we did. When we made a proposal to open up their clinic program to the entire college, we were turned down. They wanted to retain the clinic designation and keep it for themselves and so we decided at that time, in 1973, to start our own program which we called the Mathematics Clinic.

Figure 1. HMC Clinic Cartoon

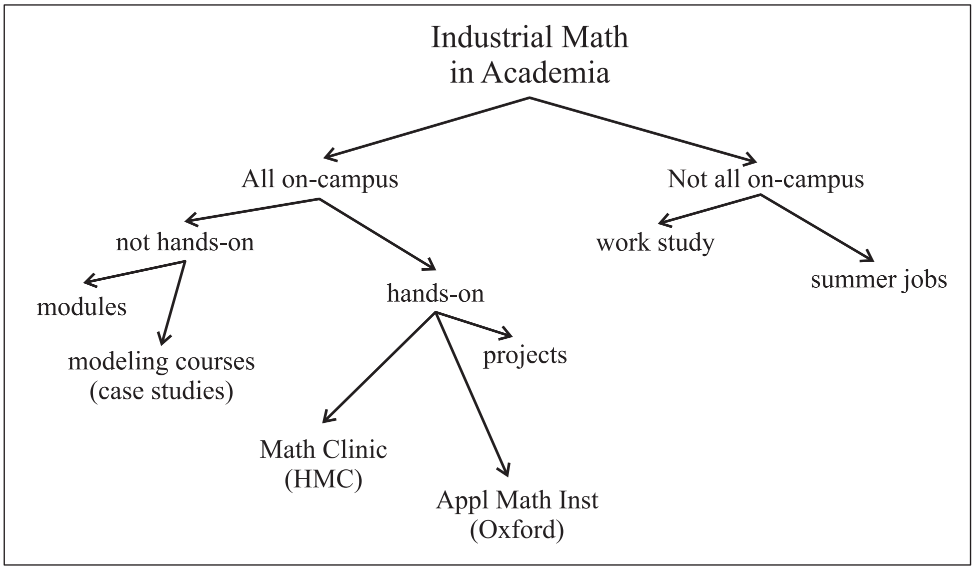

Figure 2. Classification of Projects

To everyone's surprise, it was successful. We started small, with one project at the very beginning. It was very successful, and we built on that. We had support from the Sloan Foundation and the National Science Foundation. Each of our clients is charged a fee. In 2004, our fee was $40,000 per project. One of the things we do with that money is to help support our Ph.D. program and our Master's program at the Claremont Graduate University. The Claremont Colleges do not hire teaching assistants. If you don't have teaching assistants, how do you support your graduate students? At Claremont, we do that with clinic fees which are used to provide stipends to graduate students. A few of those students work on our clinic projects.

From what I've seen, there are a number of ways of doing industrial projects in academia (see Figure 2). You can do them on campus or off campus. If you don't want to do them all on campus, then you have two options: work study programs and summer jobs. Many people opt to go in that direction because these programs are easier to manage and they do produce results. On the other hand, you may want to do the program completely on your own campus. If you do that, then there are two choices: you can either decide to have a hands-on approach or you can look at case studies. You can start a modeling course for which there are lots of good books that are basically compilations of case studies of industrial projects. COMAP [the Consortium of Mathematics and its Applications] has done a lot of that sort of thing, and you can pick up those case studies in the COMAP modules and put them into a modeling course.[1]

We decided to take the on-campus, hands-on approach. Here there are basically three ways to go. There is the Math Clinic as done at Harvey Mudd College at the undergraduate level. Or there are the applied math institutes which basically do the graduate-level applied math problems. Finally there are projects which you can do in any class. Let me describe in a little more detail two seminal hands-on programs.

The first of these are the applied math institutes started at Oxford University. These began a long time ago, but basically graduate schools still use Oxford's approach. Claremont Graduate University started this way. Now they have come back to a program like ours at HMC. The second of the seminal programs, one at the Institute for Mathematics and its Applications (IMA) in Minnesota, works differently. What happens there is that at the beginning of the semester, a number of industrial representatives are invited to meet with faculty and graduate students in a room for a week of intensive brain-storming. The role of the industry people is to present problems for discussion. Maybe something will catch fire. If it does, then

[1] See the chapter on COMAP.

the graduate students and the faculty work on the problem. Perhaps it turns into a thesis topic. It is a very intense experience for a very limited time, for one week, maybe two weeks, where everybody is put in very close contact.

We don't do that. At the undergraduate level, we spread our experience over the entire year, and we build it into our curriculum. The Mathematics Clinic is a three-unit course each semester. The only prerequisite at Harvey Mudd is senior standing. It is a required course for our applied math people. However, there are some math students who want to avoid the clinic. They have no intention of going into industry, and they don't want to work on anything that's remotely connected with the defense establishment. These students can opt out of the clinic by going into the general program of mathematics. That way they can take topology, algebra and lots of other courses and don't have to worry about applied math. But the applied math students are required to take a whole year of Mathematics Clinic.

Since I was the director of the Math Clinic, it was my job to find projects. The year began for me at the end of April. Why? Because at the end of April, we have Projects Day. This is a day set aside for presenting the results of the previous year's work. We make it into a professional meeting to which we invite lots of people. In fact, about 500 people show up. It's run just like a professional meeting with lots of talks. We do it together with the engineers so there are many people on campus. I find that this is a good time to get started, because spirits are high and the industrial representatives who are visiting see what our students are capable of. They are able to judge what a good project is since they have a chance to see a wide variety of them. That's when I got started doing my work. In 1996, for example, it turned out that I had already identified four projects for the next year, just from Projects Day of April that year.

There are always surprises. In industry, things change very quickly. Money that was there in April may be gone by September, and, in fact, a whole unit from a company may be gone. Maybe they have moved to Arizona, something which has actually happened to us. Anything can happen, so you never want to count on things until they're solidly in hand. That made my life really hard because I knew who had preregistered for Clinic. In 1996, for example, I had 13 undergraduate students who had registered for Clinic. A couple of graduate students would join in addition to these. So I had 15 students. How many projects would this support? I didn't like to put more than four students on a project. Sometimes I would stretch that rule of thumb. But I find that if you put too many students on a project, you find that one or two students do the work and everybody else is a spectator. That's not good. Also it's very difficult to manage a large team because it's hard to find free time for them to get together and do the work. The larger the team, the more difficult it is to find meeting times. Consequently I preferred three-person teams. And sometimes we could add a graduate student if that was appropriate.

Given that, it seems that I would have to go out and find five projects. But I had to balance both sides of the equation. Not only did I have to find the projects and hope they would still be around in September, but I had to worry that the students who pre-registered didn't change their minds and drop the math major to become physics majors, in which case they would do physics research. So I did a juggling act until it came right down to the wire when people were committed in September, and I could breathe a sigh of relief.

As you can see, it was a bit difficult to run a program of this kind. I did it for about 22 out of the 28 years I worked at Harvey Mudd, and it was always fingernail-biting time until September rolled around and I got a full house of projects. I never failed, but I came pretty close to not filling up all the slots.

At the undergraduate level, students received three units of credit and nobody got paid. At the graduate level, it was different. Graduate students got a stipend, and that was fine. Ten thousand dollars or whatever it took for graduate students to survive. But the rule at Harvey Mudd for undergraduates was that you couldn't get pay and academic credit at the same time. The faculty member, however, did get release time. If the course was your assignment, then that was one of your courses. We insisted on that.

Figure 3, which is representative of a year's list of projects, shows all those we had in 1996. I've listed the engineering projects as well as the math. You can see who the industrial sponsors are: Aerojet, Arco

HARVEY MUDD COLLEGE CLINIC 1995–96

ENGINEERING (partial listing)

- **AEROJET** *Design of low cost 3-axis stabilized platform*
- **ARCO ENVIRONMENTAL** *Testing oxygenating materials for groundwater bioremediation*
- **ARCO TRANSPORTATION** *Comparison of exhaust stack emissions for steamships and diesel ships*
- **AUTO CLUB OF SO. CALIFORNIA** *Remote emissions sensing from vehicle tailpipes*
- **BECKMAN INSTRUMENTS** *Blood draw tube cap identification*
- **CHIRON VISION** *Design of glaucoma pump to maintain intraocular pressure*
- **CLOROX** *Design of variable flow eductor for blending operations*
- **COBE BCT** *Automation of software testing and validation for apheresis procedures*
- **CYPRUS MIAMI MINING** *Modeling of copper smelter offgas flow behavior*

MATHEMATICS

- **3COM** *Dynamic optimal inventory modeling of finished goods*
- **EATON** *Strategic forecast for the powertrain market*
- **ESRI** *From here to there: Finding a short path quickly with limited memory*
- **GOLDER FEDERAL** *A transport pathways algorithm for the repository integration program*

COMPUTER SCIENCE

- **ELECTUS** *Electronic patient chart*
- **JET PROPULSION LABORATORY** *A graphical user interface for a uniform database interface*
- **TERADYNE** *AsciiMaster database maintenance system*

Figure 3. HMC Clinics for 1995–96

Environmental, Arco Transportation, the Auto Club of Southern California, Beckman Instruments, Chiron Vision, Clorox, Cobe, Cyprus Miami Mining, and others.

Now, what's disturbing about this picture? Well, we have one local project and three that are far away. What occurred is that the defense industry, which was very prominent in southern California and which gave us a lot of projects, dried up with the end of the Cold War. So I had to look further and further afield to find projects in mathematics. To develop projects, I looked first to our alumni, so I had to know where our alumni were. That's how we got the Eton project. There's an alum who was in charge of Research and Development for that company, and that's how we got that particular project. At first, it was difficult to find such projects, but I grew more successful year by year, and I began to know where to look for local sources. The crop of 1996 projects was different from what we had before. I would say that if you plan to get involved in this sort of work and to survive, then you've got to be flexible. It's not something that you can set up and expect it to run by itself. Things are going to be changing all the time, and you've got to follow things along.

Let me point out that at Harvey Mudd, we have two other clinic programs now. There is the Computer Science Clinic which we established in 1994. They had three projects, from Electus, JPL and Teradyne. There is also the Physics Clinic that's just started. That's too bad from my own point of view because I used to get some very good physics majors to work on Math Clinic projects, and now they aren't there any more. There are even rumors of a Biology Clinic. It's my opinion that we're going to have a *de facto* College Clinic, whether the engineers want one or not.

I did consider an interdisciplinary clinic once myself, but it involved a turf battle. Still, it's what one should do, but you can't fight city hall, so we just did what we knew how to do for ourselves. Maybe the clinic idea will evolve into interdisciplinary work. Once every department has a clinic, somebody may get the idea that there should be just one clinic, and it should be interdisciplinary. But I'll let somebody else work on that.

This gives you an idea of the size of the program at Harvey Mudd. It's huge, and there are many students involved in it. How are students prepared to enter this program? After all, they're just undergraduates. By the time these students become seniors, are they really ready to work on open-ended interdisciplinary projects? It comes as a surprise to a lot of people that our students can actually do these things, but they can, and there are several reasons for this. One is that once you establish a program and it's in place, then younger students have two or three years to see that program in action. They go to all the talks, they hear the scuttlebutt in the dorms, and they know the ups and downs and what you have to do to prepare yourself to work on a project. By the time they're seniors, they have the right mind-set to work on projects. That's one thing. The other thing is that, at Harvey Mudd, we start preparing our students to work on open-ended projects early in their careers. We start with the freshman year with an engineering design project which is not required of everybody, but a majority of our students take it. That is where they learn certain skills for managing a project.

At Harvey Mudd, one reason why the clinic projects turn out well is the way the Clinic is organized. Ultimate responsibility for the Clinic rests with the Board of Trustees through one of its subcommittes called the Clinic Advisory Committee. I, as well as the directors of the other clinics, sat on the committee. In addition to us, there are about a dozen people from industry who are high-level R & D people who advise us on a number of matters, not the least of which is the fee that we should charge. We have two trustees who come to every meeting of the Clinic Advisory Committee. They also show up once a month for an on-campus meeting. Every Tuesday at Harvey Mudd, there are clinic talks. There are progress reports, and the Advisory Committee people go to and listen to all the talks. Then the team leaders attend a lunch meeting at which the committee members offer advice on how the talks went and how they could be improved. The committee members really take an interest in these things. Of course, the Dean of Faculty is the real boss for all the clinics at Harvey Mudd, and clinic development is this person's responsibility.

Understand that, as director of a clinic, I was not totally alone in going out and scrounging for contacts. We have two people at Harvey Mudd whose sole job is to look for and help us with projects. But from a certain point on, it was up to me to clinch a deal. Once I got an introduction to somebody, I took it from there. The clinic development people work at the corporate level, and they have contacts within the corporations themselves. Sometimes clinic projects are funded by corporations. This is always good because that means the money doesn't come out of somebody's budget. In any case, however, I got leads from the development people and the advisory committee. They made an introduction for me and gave me a phone number so that I didn't have to make a cold phone call. The worst thing in the world is to make a cold phone call. That's not the way to develop a project. You want to have somebody at a high enough level to introduce you to a potential client with a budget to support a project. That's the best way to get started. The rest is up to you.

Refer to the organizational chart in Figure 4. Note the placement of the directors, the Math Clinic, Engineering Clinic and all the other clinics. In fact, this is already out of date. We have administrative assistants for the directors, and further down are the teams. A typical team, at least for us, looks like this: We have the faculty advisor and a representative of the client whom we call the client liaison. I would not take a project in which the sponsor did not provide somebody as a direct contact because then it was no one's problem, and that was a sure way to fail. There's no way you can satisfy the client if you don't stay in touch with what's going on at the client company. So I needed to have assurances from the client that there was somebody who would show up on a regular basis and act as a member of the team. If the client was far away, as in Michigan, then you had to be creative. What I did in that case was to buy a package

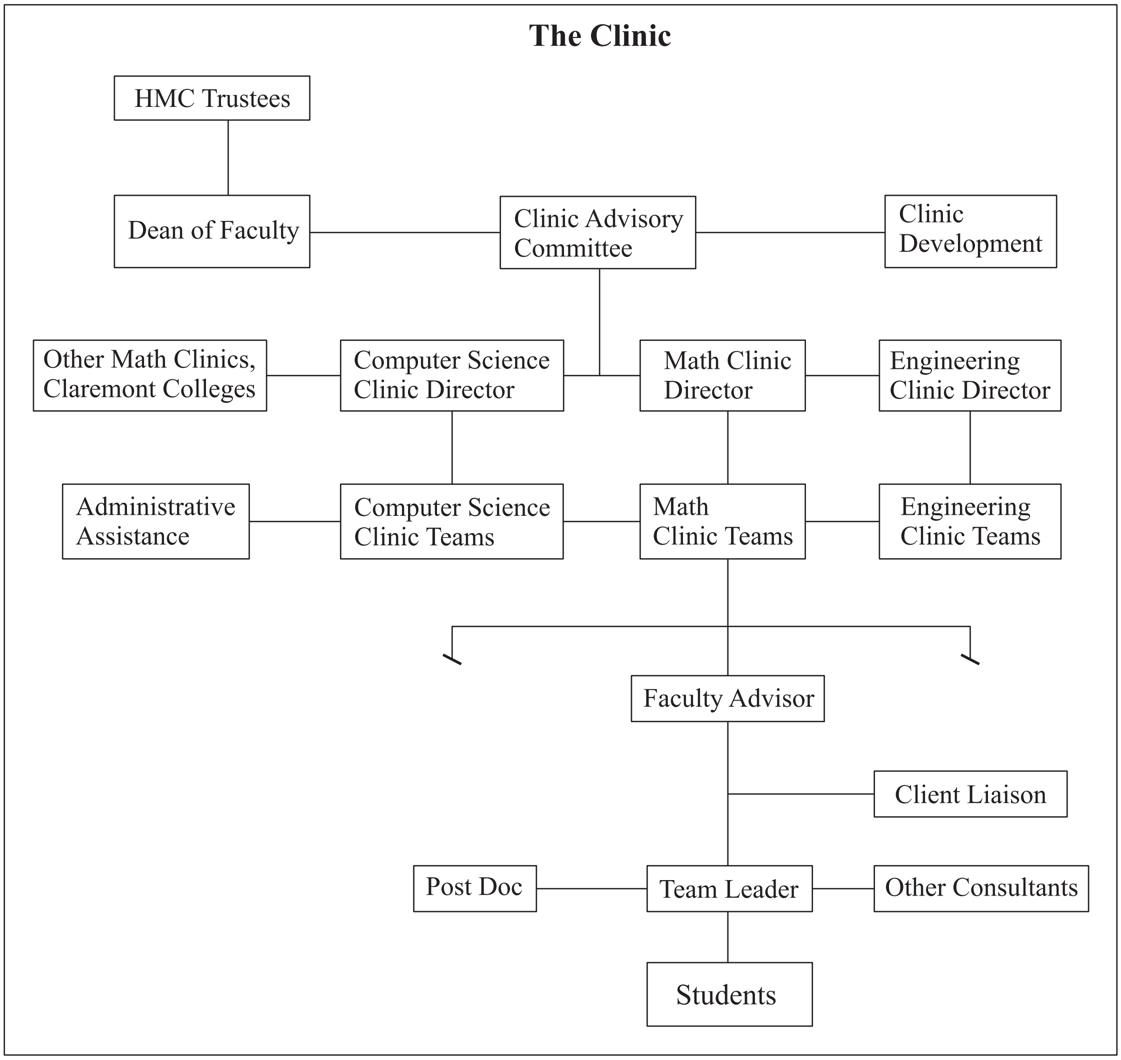

Figure 4. Organizational Chart

called ShareVision, which is a little package with a TV camera which you hook up to your PC. Connect that to the phone line, and you can do video-conferencing. Then you can have face-to-face contact with your client even though the company is very far away. Unfortunately this didn't work too well. We started doing this in 1995–96, but for some reason we encountered some resistance. Certainly it was somewhat impersonal. Maybe it will catch on later, or maybe we won't have to do it at all. Nonetheless, it's necessary to have somebody from the organization intimately involved with the project, preferably somebody who is interested in the result.

The worst thing that can happen is to have assigned to your project a liaison who doesn't care about it (See Figure 5: The Role of the Client Liaison). It's just another duty, one more thing that this person has to do. That is a very bad situation, and I would not take on such a project.

An important detail that we have built into our program is the selection of the team leader, which is not done democratically. The teams themselves are not formed democratically. Let's say that I've got thirteen students, a couple of graduate students, and four or five projects. How do I assign students to projects? What worked for us was this: the faculty advisors and I would sit in a room, and we didn't come out until the assignments were made. That's hard, you know, because everybody wants the good students. We're a small school, and we all know who the good students are. Interestingly enough, it's not advisable to have

The Role of the Client Liaison

Along with the faculty advisor, the client liaison plays a very important role in advising and evaluating the ongoing work of their clinic team. To all intents and purposes, the liaison is regarded as a crucial member of their own project team.

Project work statement. The preliminary work statement was drafted with no input from the student/faculty project team and often contains language which is vague or too open-ended. For this reason, the first order of business for the team is to come up with a work statement which realistically takes into account the sponsor's objectives as well as the time (or other) constraints on all team members (including the liaison). Here the client liaison plays a critical role, but it is the student team members who draft the "official" work statement for the project. This work statement should address the following points:

- The project's objective or expected results
- Description of how the team will achieve the objective
- Equipment and software to be used
- Anything that might affect the budget for the project

Communication. Regular communication between the liaison and the team is crucial for the success of the project. The liaison conveys the client's needs and usually serves as a source of information and a technical authority. For example, the liaison's suggestions might keep the team from investigating alternatives that have already been tried unsuccessfully.

Team interaction. Experience has proven that interaction between the team members and the client liaison is critical for the project's educational value–as well as a successful solution to the problem posed. This interaction provides students with the experience of working with people who have to meet goals and get things done by a certain time. Moreover, it gives them a feel for the real world of work and the problems of meeting a budget or a deadline.

Figure 5. The Role of the Client Liaison

only good students on one project. For one thing, personnel problems can arise, and they usually do when you have two very strong-minded people on the same project. I can tell you now, after all these years of experience, that we've never, ever had a project fail because of technical difficulties. We've come close to failure twice because of personnel problems. They're going to happen. But it doesn't matter. Currently in industry, the same thing happens. In fact, it is a common occurrence. You know when you're assigned to a project with other people that there are going to be some fights. Strong-willed people will duke it out. But you know, somebody has to give in eventually, and you've got to work out your differences. That's part of teamwork and part of what our students learn at Harvey Mudd. Incidentally that's something that doesn't come naturally to us mathematicians. We are loners for the most part. Right? We sit in a room, and we do things which eventually we share with somebody. We may tell a person or two, but we're not used to working in teams and capitalizing on the strengths of others while acknowledging our own weaknesses. God forbid that we should do that. But we *need* to do that. You need to be brutally honest, working in a team. You have to learn that you don't cover up. Don't defend mistakes or stupidities. Own up to your own mistakes and get on with your life. You have exactly two semesters to finish your project by Project Day in April. Your report has to be well under way, and you've got to have the thing in the can by that time. Personnel problems notwithstanding, just work things out. My problem as director was to see when these things were going to happen. The way I did that was to have bi-weekly meetings with the team leaders. As I said, team leaders are not democratically chosen. At the time we form the projects, we choose the team leaders. I don't mean to imply that we're dictatorial here. We do ask the students first to give us their preferences. A list of projects is sent out with as much detailed description as possible at the time. Students are asked to rate them in order 1, 2, 3 whereupon we try to give people their top rating. Actually we can do that most of the time, but we can't do it all of the time. Sometimes students are given their second choice, sometimes their third choice, and that's hard.

The team leader is a crucial person. He or she need not always be the strongest person on the team. In fact, it may actually be wrong to make the strongest person on the team the team leader. Every two weeks I had a meeting with the team leaders. We went around the table and talked about how things were going, and we tried to predict whether there would be some problem coming down the pike. For example, the client had a certain sort of platform and a certain sort of software which we didn't have. How were we going to get around this? Fortunately we now have a full-time computer intern in the department. He attended our meetings so that we could jump on these problems immediately and solve them before they threatened the success of the project.

We get visitors at Harvey Mudd and at Claremont Graduate University who want to learn more about the Clinic. In fact, I invite anyone who would like to know more about the Clinic to take a sabbatical leave at Harvey Mudd. We can put him or her in a clinic project as a post-doctoral advisor. As a post-doc in a clinic project, you can learn a lot about the Clinic and how it works, and then go off and do something similar yourself. It's much better to have hands-on experience with the Clinic than to hear about it in a forum.

Figure 6 shows our advisory committee for 2004 with an affiliation for each of the committee's members.

CLINIC ADVISORY COMMITTEE, 2004–2005

Mr. Paul Anderson. The Aerospace Corporation Mail Station M1-136, 2350 El Segundo Boulevard El Segundo, CA 90245

Mr. Michael Bell. Advanced Tech Center Beckman Coulter MS W-355, 200 South Kraemer Boulevard Brea, CA 92822-8000

Joseph Betser, Ph.D. Senior Project Leader Business Development, The Aerospace Corporation Mail Stop M1-102, P.O. Box 92957 Los Angeles, CA 90009-2957

Mr. David Lesyna. Vice President, Engineering Optivus Technology, Inc., 1475 South Victoria Court San Bernardino, CA 92408

Wah Lim, Ph.D. Trustee, 30 Canyon Fairway Drive, Newport Beach, CA 92660

Mr. Ken Livak '74. Vice President, Science Applied Biosystems, 850 Lincoln Centre Drive MS 408-2 Foster City, CA 94404

Mr. John H. Livingston, retired. 31 Augusta Lane, Newport Beach, CA 92660

Stephen J. Lukasik, Ph.D. 1714 Stone Canyon Road, Los Angeles, CA 90077

Mr. John B. Mayer. Director, Hard Copy Technologies Laboratory Hewlett Packard, 1501 Page Mill Road Mail Stop 2U16 Palo Alto, CA 94304

Mr. Douglas Overland '95. Research Automation Engineer II Amgen, Inc. Mail Stop 29-1-A 1, Amgen Center Drive Thousand Oaks, CA 91320-1789

Mr. Robert J. Prodan '71/'72. Chief Technology Engineer Guidance, Navigation and Control, The Boeing Company, 3370 Miraloma Avenue MS CA-63 Anaheim, CA 92803-3105

Mr. Antony N.W. Selim '97/'98. Senior Systems Engineer and HMC Campus Manager Command, Control, Communication And Information Systems, Raytheon Loc. FU, Bldg. 675, M/S Z233, 1801 West Hughes Drive Fullerton, CA 92833-2200

Mr. Craig Snow. Engineering Manager Advanced Systems, Northrop Grumman Space Systems Division 1100 West Hollyvale Street, P.O. Box 296, Azusa, CA 91702

Mr. Carl Spangenberg. Director, Irvine Ranch Water District P.O. Box 57000, Irvine, CA 92619-7000

Mr. Cary Talbot Sr. Systems Engineering Manager, Medtronic MiniMed, 18000 Devonshire Street, Northridge, CA 91325-1219

Mr. Nywood Wu, CEO, Globe Plastics 13477 12th Street, Chino, CA 91710-5206

Figure 6. HMC Clinics Advisory Committee

Dear XYZ:

We welcome "company name (CN)" as a sponsor in the Harvey Mudd College Mathematics Clinic program for the period, September XXXX to May YYYY. This Letter of Understanding is intended to set forth the agreed-upon operational details concerning work on the CN project in our Clinic program. The project will be more explicitly developed through contact between our faculty and CN but is generally described as follows:

This research shall be under the supervision of faculty member(s) appointed by the College; however, the actual research shall be conducted by three or more students of the College. It is understood that you will appoint someone from CN to act as liaison between the team and CN. All College facilities are available to our Clinic teams: libraries, computers, clerical, etc. It is understood that neither the College, the students, nor the faculty shall in any way assume any liability for any damages whatsoever which may be incurred or sought in connection with the use of the product of the research program.

It is understood that CN will assist in financing the XXXX–YYYY academic year research program by paying the College the fixed sum of $40,000 which will be used to defray the costs incurred by the College in connection with the establishment and supervision of the program. The funds are normally paid to the College in three equal installments, the first at the beginning of the fall semester, the second at the beginning of the spring semester, and the third following receipt of the final project report. CN will be billed at the appropriate time. If this arrangement is inconvenient, then other arrangements can be worked out.

The College understands that information that is received in connection with the performance of its obligations to the project will be treated as confidential and will not disclose any confidential information without your express written authorization. A copy of our "Policy on Confidentialities" is enclosed. The College further understands that all inventions, discoveries, and improvements which are made as a result of the research program shall belong to you and any background information relating to such matters shall be made available to you upon request. The College will cooperate in protecting your interests in any inventions and shall further provide you with information necessary to perfect any claim you may have with respect to the ownership of such rights, including providing you with an assignment of all the College's rights in and to any such invention.

The College does reserve the right to publish papers and reports concerning the research program in accordance with its publication policies. No such reports shall be published without your approval within one year following the completion and delivery of the final report. After that year, you will be given an opportunity to review and comment on any proposed publication.

The research program is to be conducted by the College to benefit its students and faculty and, at the same time, to benefit CN and ultimately, the general public. It is understood that the College is engaging in this activity in furtherance of its educational purposes and that it is not intended that the activities of the College are to be a type ordinarily carried on as an incident to commercial or industrial operations.

If the foregoing meets with your approval, we would appreciate your signing a copy of this Letter of Understanding and returning to the undersigned.

Figure 7. Letter of Understanding

I believe that people should start programs of this kind. In doing so, they can learn from the engineers in many ways. Be as professional as possible. For example, have a clear idea of what your policies are. You should know as precisely as possible what your relationship to the client is, and you should be as professional as possible in that relationship. At the very beginning, we had a letter of understanding that I drew up and sent out to each of our clients (See Figure 7). That, in effect, was our contract. We didn't call it a contract because the clinic is an educational program, a subject to which I'll return a little later. What we do is not R & D. Industry may be tempted to view a project as R & D, and I have to keep reminding people that failure is a possibility because the Clinic is an educational program. You don't want to tie an educational program up in knots with lots of bucks and contracts and so on. The basic reason for our letter of understanding was to clarify how the program would operate. And the first detail, of course, was a general description of the problem. In the early stages, you probably won't have a clear idea of what the problem is. That's *why* it's a problem, and you've got to work it out. So your first job, when the project starts, is to develop, in collaboration with the company liaison, a clear idea of the problem and to produce a couple of paragraphs to describe it. Then you lay down a timetable for meeting the goals which a solution of the problem entails. I would put forth a general problem description with the remark that the details would be worked out later. That was the team's problem: to produce a specific description with a timetable for finishing the project.

At this point, we came up with a number. We expected that the client would help defray the cost of the program. It was agreed up front that anything that we developed belonged to the client. We promised to keep records so that if anything looked as if a patent application would be needed, we had enough data to provide our clients with what was required of them to do that. We had a policy on confidentiality which we gave them in writing, promising not to talk about things that we shouldn't talk about. Our clients needed to tell us what we shouldn't talk about, and we respected their wishes. This was something everybody had to learn in industry, anyway, so why not trust college students with confidential information? We agreed not to publish any papers without permission for the first year. After that, we could publish papers on any of our material, but we gave our clients the right to read the text first in order to correct any false impressions or inaccuracies. Nonetheless, in the first year we agreed not to publish anything without their permission. The reason for this was the following: if clients couldn't capitalize on our work within a year, then it probably didn't matter after that whether they did or not. Other people were working on similar ideas, and if you had a year's lead and couldn't do anything with the results of the project during that time, then you probably didn't have to worry about confidentiality afterward.

There's something else which is really important. The research program was to be conducted by the college to benefit its students and faculty and at the same time to benefit the client and ultimately the general public. It was understood that the college was engaging in this activity to further its educational purposes and that it was not intended that the activities of the college be of a type ordinarily carried on as an appendage to commercial or industrial operations. We had to say that because, as you might surmise, you could very easily disappoint clients who thought they would get one thing and then got something else. We didn't want to have unhappy clients, so we liked to put those things up front. At the end of the letter of understanding (see Figure 7), we stipulated that it be signed and returned if the foregoing met with the client's approval. That document started the clock on the project. We assigned students as soon as I got this back, and then we began the work. The letter of understanding was the only legal document that we used in the Clinic, and the wording of this document was obviously done in collaboration with the college lawyers.

Figure 8 shows a timetable for a typical project. Who uses such a table? We all do. It's a standard document for all of us. Once the teams get underway, one of the first things you do as a professional is to develop a timetable which you update at every meeting. You would think that everybody could remember what's going on from one meeting to the next, but keeping a timetable like this is very important for the team leader who does not want to be in a position of being the boss who lays down the law. When you have a timetable like this, it's the timetable that everybody buys into. That way you don't have someone coming around to order you to do something. This is perhaps a small but crucial consideration psychologically. Once people agree to a timetable, then it's understood what is going to be done and when it's going to be done. If something is not going to work out anymore, then you modify the timetable. The timetable becomes your guide, your friend. You take that attitude all the way through. If a team leader ignores this, then you can guarantee that there will be problems later on. The figure shows a generic time table in the sense that this is the kind of thing that you can write down without even having a project. That's how you start, and you can see how it goes. There has to be some kind of a literature search in the beginning. You've got to look around and see what's out there in terms of relevant papers. You've got to consider how to define the problem before you can set out to solve it. Of course, all this goes into the final report. Notice how you start writing the final report as the project progresses. That's something that people don't think about, but early in the year, you should start with parts of that final report like the introduction. You

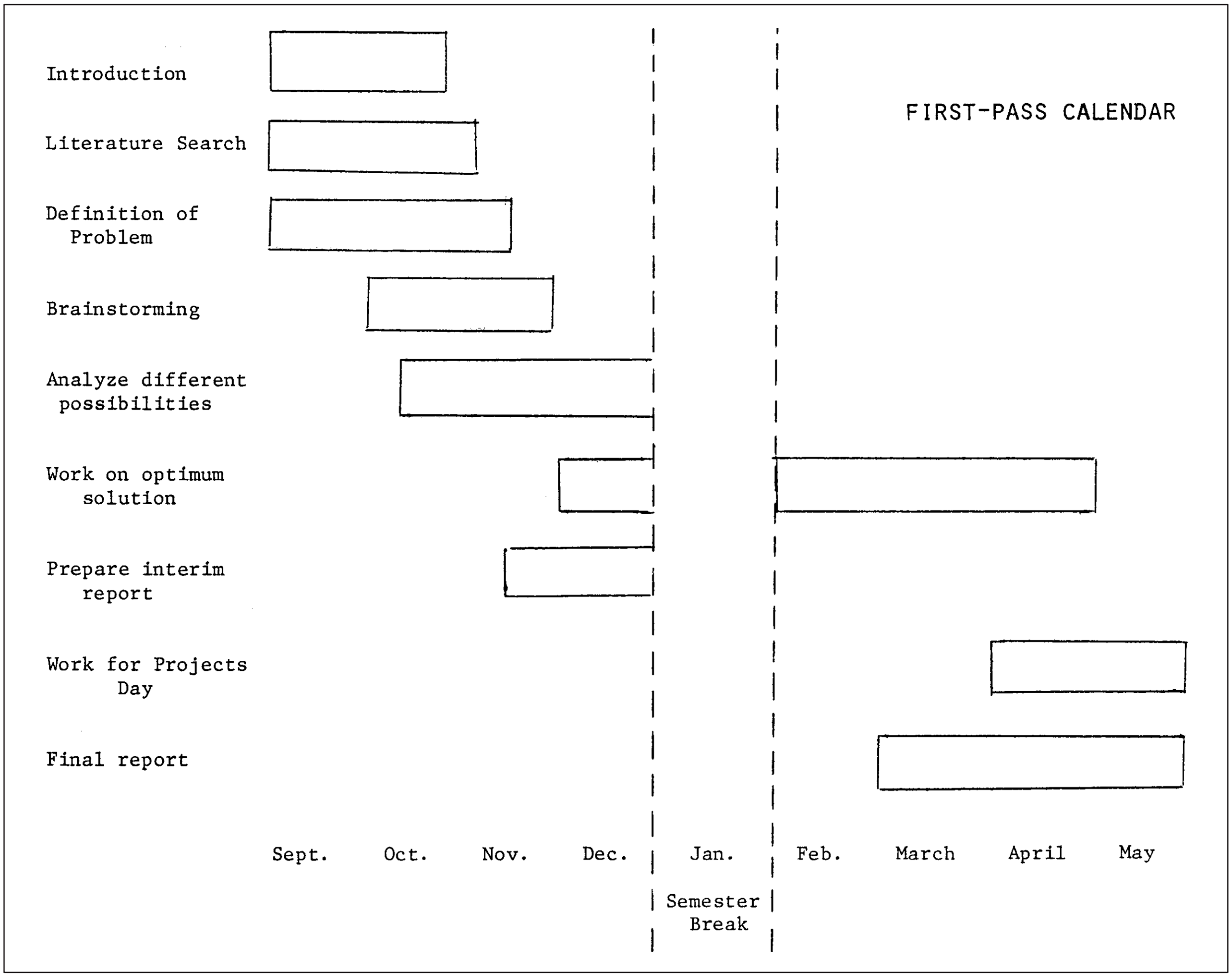

Figure 8. Schedule of Activities

don't have to wait until the very end to start writing your final report. That's another advantage of laying out a timetable like this, so people can anticipate what's coming down the pike.

I like to develop a staffing sheet [See Figure 9]. A clinic secretary did all this, and she was very good at it. Each project has its own staffing sheet which every member of the team gets, so everybody knows who all the players are. We have telephone numbers and e-mail addresses for the liaison, the faculty supervisor and every member of the team. At the bottom of the staffing sheet are the team's meeting times. A copy of this goes to the client and the client representative because this person is a member of the team. Everyone on the team can contact any one else on the team. Again it's part of being professional. When you put things down on paper like this, all the players know who they are and what their roles are.

We helped students orient themselves with a handout called the Math Clinic Student Handbook. Written by some of our own students, it told them what resources they had available at the colleges: the computing resources, the library resources, and how to write reports. All of our reports are written in $\TeX$, so we expect our students to learn $\TeX$. We could use something else, but we preferred $\TeX$. In addition, there was some information on how to design your final report. With all this information at their disposal, the students finally got underway.

Be professional in client relations. That's the Number One Rule. The Number Two Rule is to have stopping rules: know when you are done. If you don't know when you're done, you and your client are both going to be very unhappy.

HMC Mathematics Clinic Fall 1995

Client: ESRI, 380 New York Street, Redlands, CA 92373

ARC/INFO, a product of ESRI, Inc. of Redlands, California, is a software development tool for GIS (Geographic Information Systems) applications that modify, analyze and display geographic information. An important feature of ARC/INFO is its ability to find the shortest path between two points in a road network. ARC/INFO currently employs Dijkstra's algorithm which is known to be provable optimal, but performs poorly with large datasets due to memory constraints. If the requirement of finding an exact solution is relaxed in these situations, heuristics can be used to compute approximate solutions.

The goals of the Clinic team are to investigate various heuristics for finding the shortest path, paying particular attention to the proposed "falling double-tree" algorithm; to determine criteria under which each method performs most effectively; to develop and test code using Network Engine, ESRI's library of low-level routines for managing geographic information.

Clinic Participants

Liaison:	Dale Honeycutt (Office) (909) 793-2853 E-mail: dhoneycutt@esri.com
Faculty Supervisor:	Professor Lisette de Pillis ext. 18975 E-mail: depillis@hmc.edu
Clinic Team:	Darryl Yong (Team Leader) ext. 74752 E-mail: dyong@hmc.edu
	Tu Banh ext. 74714 E-mail: tbanh@pomona.edu
	Steven De Neefe ext. 71430 E-mail: sdeneefe@hmc.edu
	Richard Krabill ext. 74780 E-mail: rkrabill@hmc.edu
	Andrew Ross ext. 73442 E-mail: aross@hmc.edu

Meeting times: Monday 12:00 (noon) – 1:00 pm, East PDR, Platt Campus Center, Thursday 11:00 am, Mathematics Clinic Room, Olin 157

Figure 9. HMC Math Clinic: sample client

Don't expect the client to always follow your advice.[2] This is meant especially for mathematicians who have a really slick way of solving a problem. Be aware of the realities of student commitments. There are all sorts of activities on campus, and occasionally students want to do something other than work on their clinic project. You will have to be sensitive to this and make the necessary allowance for it.

Beware of software projects. I realize all projects eventually come down to producing software at some stage, but if they start off as modifications of somebody else's software, this is deadening. There's a lot of money out there for that, but I would stay away from such things, especially one-semester projects. In fact, avoid half-year projects like the plague. It takes you half a year just to understand a problem. If you take a problem that involves only half a year, you're just getting into the project when it's all over. I would recommend only year-long projects.

[2]See the chapter on Project Deliverables I.

Expect personnel problems. Hope that they will be small ones, but they'll be there in any case. Adhere to the stated confidentiality policy. I know that it's a temptation to go and tell people that you have a nice result, but you have to be careful to observe that policy. Then this last caveat: not everyone agrees with me on this, but I've been burned before. Control your own hardware/software environment. When I first started the clinic, the college had a central facility. This involved time-sharing on a big central computer. Everybody would log on and share computer time. Not being in control of our environment caused a lot of problems. We became diversified, we distributed computing and workstations, and we controlled our own environment. I bought my own machines. They were not all that expensive, and I did have some resources. We also had a computer intern full-time. So I was in control of my own computing environment, and I avoided those problems that I had before. I realized that when you're first starting out, you can't necessarily do that. But try to work in that direction. That, anyway, is my advice to faculty getting started on this sort of work.

Until his recent retirement, **Robert Borrelli** worked at a prestigious institution which focuses on mathematics, science and engineering. All of its faculty hold PhDs or terminal degrees in their fields. The student enrollment hovers around 700.

Panel Discussion Following "Industrial Contacts"

How are projects undertaken by students, as described in the preceding chapters, perceived by their clients? To gain some insight into their point of view, the editor invited several representatives from various industries to talk about their own perception of student projects. An edited version of their comments is given below.

Participants and their affiliations are as follows:

Mark Kopczynski [MK], Allen Bradley Company
Dirk Wilken [DW], Harley Davidson Company
Dan Guaglianone [DG], Abbott Laboratories
Martha Siegel [MS], Towson State University
George Corliss [GC], Marquette University
Robert Borrelli [RB], Harvey Mudd College

MK: One of the most important issues to consider is communications. That is probably what's going to get you into trouble over the long term. Patience is required in view of the activities in the follow-up because the project is unlikely to be an instant success. That needs to be stressed to the client.

What we learned at Allen Bradley was the importance of the team concept. This may relate to the perception that mathematicians are not game-oriented people. I'd recommend that you familiarize your students with the team concept: who the players are, what roles they play, and even to identify personalities on the team. That way you can account for why people react the way they do. We did that at Allen Bradley just to identify teams and what roles people play on them. We assume that the team leader should be doing certain things although the team leader may not understand everything that's going on.

From an industrial point of view, one of the key questions involves figuring out how industry and the academic world can complement each other's strengths. There's a lot of knowledge out in the academic world, and we have to put that knowledge to work in the industrial arena. There's also a vast source of knowledge which you can access through on-line databases and resources like the Web. That, in effect, involves communications skills. We're doing a lot of such things at Rockwell Automation[1]. The problem is that there's not a really good conduit between the academic world and the industrial world, one that can explain how we can collaborate and fuse our joint energy.

DW: As someone who works at Harley Davidson, I have thought about how we actually make industrial contacts, something which would be of value to people interested in this subject. I thought that Bob Borrelli's comment with regard to tapping into your alumni base was particularly pertinent. That, I think, is a good source. It is also helpful when looking for consultancy projects to find out what your potential contacts read. For example, if your competence is in the transportation area, find out what transportation people read. If you're strong in engineering, find out which periodicals engineers read. Perhaps you can publish some of the work that you have done in a one-page article in an industry rag. As an industrial

[1]Rockwell International now owns Allen Bradley.

person, I've established contacts just going through the free magazines that come across my desk. What I find there is often interesting, like articles which have a bearing on some of my problems. Too often in the academic environment, there is a tendency to go to a math journal to publish. Unfortunately those journals don't reach the industrial side of the house as often as some of the industrial journals which, ironically enough, are always looking for something to put on their pages. A simple one-page article on a really successful project might generate a lot of interest among people whom you might not otherwise be able to access.

DG: Let me build on a comment made by the first two gentlemen from an industry perspective. I'll start by saying that Abbott Labs is a very large organization. We probably have about 50,000 employees worldwide, 17 or 18 manufacturing locations in the U.S. as well as locations in 33 foreign countries. We're all over the place. We've got some of the types of projects described by Bob Borrelli, but they are part of our corporate culture. There are reasons why this is so, but first I have to pass on an anecdote.

When I told my wife last night that I was participating in this panel, she said, "I have a story that you can tell tomorrow." She went on to say, "We had five environmental engineering students from a state institution come in. They were going to do a project in the chemical division of the company where I work. Their goal was to analyze the waste stream from start to finish for pollution prevention purposes."

At one time, my wife was the manager of the corporate environmental division at Abbott. This group of engineering students made arrangements to do its project through one of the divisions at Abbott. My wife got a phone call from someone to inform her, in effect, that they didn't have time to babysit these students doing this project.

That summarizes the problem as I see it. There's a spectrum of modes of collaboration. We do different things with different universities. At one end of the spectrum, we may be partners in doing research, contributing money to a university or a department to fund a project. At the other end of the spectrum, we may be giving money for philanthropic purposes to a university to develop its curriculum. Somewhere in the middle fall these projects. In our case, there is no slot in the spectrum for this kind of work.

One of the issues that people always raise and one which I think is probably something of a red herring is the issue of confidentiality. We're in the pharmaceutical business, and confidentiality is a huge issue, so huge that we still didn't have a home page in 1996 for fear that somebody was going to break into our computers and steal all of our pharmaceutical secrets[2]. That's one issue student consultancies have to contend with. My current position at Abbott Laboratories is in college relations and recruiting. Previously I had served as director of health and safety worldwide for the company. So I've seen every manufacturing operation, and I know what Abbott operations are like. We manage to cope with the confidentiality issue, but this remains one of the first subjects that we broach when someone approaches us with a project: confidentiality. Nonetheless, I think that's one thing that can be worked through.

Another issue is the time required to work through projects like these. Most of our managers perceive them as educational endeavors. The mind set is that there is nothing students can tell us about running our business. So they are viewed as an additional burden placed on managers, who are already very busy. Let me cite an example of a project and the way the final result was assessed by the manager in charge. Like all the major pharmaceutical companies, we use solvent-based coatings for pharmaceuticals. The problem is that you have VOC (volatile organic compounds) emissions going out the stack which the EPA measures and which you're trying to reduce. One of the obvious remedies that you can try is to go from a solid base to an aqueous base solution. Now anybody who knows anything about the FDA knows that the paperwork involved in making such a change will be horrendous, and it will take two years to get it approved. So essentially the project the students were given was to determine to what extent our organic emissions released into the air could be reduced. The principal recommendation which came back was to convert from a solvent base to an aqueous base. Of course, we knew that. The manager in charge had said

[2]Abbott Laboratories established a web page at www.abbott.com shortly afterwards.

that we knew that. *Doing* that is the problem.

Here is a bigger concern which I hadn't really thought about, but which was clearly the concern of the manager who said he knew that the solution would be to get an aqueous base as opposed to the solvent base. The problem lies in ironing out all the details like doing the paperwork, getting the necessary funding, identifying the compounds to be exchanged, all that sort of thing. Obviously he couldn't do that. What he realized, after having gone through all this, was that he had a real problem, which these students probably didn't have the time to solve. So maybe one shouldn't be looking at these projects as problems to be solved. Perhaps such projects should be viewed as college relations projects, where we can develop relationships with departments and students, get our name in front of people, and ultimately end up with better full-time hires for Abbott. Maybe we should quit trying to convince people that this is going to involve real problem-solving. In any event, that was the manager's mind-set, for whatever reason. Perhaps we should lower the level of expectations that students are going to discover a whole bunch of new things. That will lead to developing a better relationship with whomever we have as partners on a particular project. Such an approach might help to sell this kind of project.

GC: Abbott Labs employs quite a few of our students, and I hope that you will not do that. Bob Borrelli just gave one reason: if you view the students as if you're doing them a favor, then you're not really treating them like a real consultant. Bob alluded to having difficulties if his teams are not taken seriously. That doesn't work.

DG: Just to follow up on that a bit, I do believe that there are benefits in programs like these. I believe if I dug hard enough and long enough, I could find an operation where someone partnered with a local university and actually came up with something that was helpful. If I searched, I could find them, but I think that's probably the exception rather than the rule right now. I suspect that there are companies other than Abbott which probably feel the same way. But if I could open the door a crack so that people might see some of the benefits in such a program, then it would grow. It could snowball on its own. The problem now is to get the door open.

MS: I think that the role of the director is to come in, take a problem and whittle it down to the right size so that there isn't any misunderstanding about its scope. We did a project for Vector Dickenson, a company which does biotechnology all over the world. It had a problem with a small operation in its Hunt Valley facility involving the production schedule for plated media (which are those little plates that are treated with special substances). The company had an assembly and production line with only one person trying to schedule it year round. It's a complicated problem which involves inventory, quarantine considerations, and cleanup problems. This project actually was extended to a second year. Now there were a lot of inputs to this problem. In one sense, this sounded like a software project, but it was really an operations research project. I think the company got far more than what it paid for, but that was because we said "Okay, this is what we can do for you," and we did it. It may have sounded like a small problem to the manager, involving one small area of scheduling, but it was more than she could have done alone.

MK: I think what you're alluding to is that you really have to avoid problems like world hunger which require an enormous industrial team to approach. You're setting yourself up for failure because the scope is too big, as Bob Borrelli said earlier. I know from Allen Bradley's standpoint that we have used students in this capacity, but it's a long process. Just last semester, we had a student through the Milwaukee School of Engineering (MSOE) who was an engineering and statistics student. He was given a problem we had in our molding area, where the issue was to design an experiment with certain parameters and determine what activities most effectively controlled the results. This engineering student published his normatory test results. The rule at Allen Bradley is that the student we hire has free rein to come in at whatever hours fit his or her schedule and our timetable. You have to be flexible but you really need to stick to your mission statement. It may sound like a lot, but once that is done, the payoff can be great. In this particular case, Allen Bradley reaped a $250,000 savings from the student's work. We may not be talking world hunger here, but a $250,000 savings at Allen Bradley in one particular area is an impressive savings. That's the payback in which the student won, and Allen Bradley won. It became a win-win learning experience.

DG: According to *Princeton Review*, Abbott Labs has one of the highest rated internship programs in the country. In some ways, this surprises me because I'm not satisfied with our ability to retain our interns as full-time employees. I think, in fact, that our retention rate is low compared to the industry average. What the *Review* said about our ranking is that all the students interviewed had offered the opinion that they did real work at Abbott.

We just launched a new-age drug, and there are probably five or six interns who worked on this last summer and who finished the clinical trials. If I have a project like the one Mark just described, it would be something I would hire interns to do, not a team. When interns come on board, they are Abbott employees for three months. What's involved here may be a control issue. I'm not sure, but we don't even think about these opportunities as an option for solving problems with a team. I'm really interested in alternative work patterns, and maybe someone can suggest how these can be implemented.

Comment: How does downsizing affect matters? Lately there have been a number of situations in which there aren't enough full-time employees to handle some of the problems that companies have to deal with. These companies are just sitting on those problems. They're not crucial enough to justify hiring somebody. So sometimes this kind of problem is ideal for student projects.

DG: This goes back to Professor Siegel's point, doesn't it? Let me speak to the Lake County operation where we have probably 14,000 to 15,000 employees in a manufacturing environment as well as an R & D setting pretty much like what you see throughout the world of Abbott. As I think back now on the few projects that we've tried to do, the scope may have been unrealistic. Yet my perception of this issue has been formed from these projects; however, I know that if I went to some of our off-site locations, I could find projects where we've partnered with schools and universities, and they were successful. It may just be an issue of scope.

MK: The other issue is sharing your successes and celebrating the things that have been done. When you get the fanfare, other parts of your organization will take note. Accolades will come to the students, and full-time folk at Allen Bradley will be recognized for having brought in a resource. In addition, the person who did the work now has a known track record at Allen Bradley if he or she chooses to work there.

Q: What about summers? Have any schools tried using the summer time instead of the regular term-year?

RB: Our students can't do that because they use summers to make as much money as possible.

MS: It takes so long to understand the problem and in our institutes, we found that data which are promised are not always delivered. It takes industry all summer to delineate the problem. We have two faculty reps who go into the industry during the summer to try to understand the problem. They do all of the background reading they can so they can pick out what the students should read in the fall. They use industry clean-up records, purge files that have to be disposed of in any case, to get things in a form that they can use. So I don't think it's too practical from a team standpoint to work on projects in the summer months.

DW: There's another issue on the industry side of things as well. What do you think the highest vacation month loads are in industry? June, July and August. So the student's chance to work with a regular employee is going to be limited in the summer. From my standpoint, it would slow things up.

DG: What about co-op arrangements? We worked our way out of these, particularly in the engineering disciplines. A lot of the engineering schools and their students are in co-op programs, but over a period of time, probably more for the sake of convenience than anything else, we've gone to summer blocks where we employ interns. We continue to have some co-op programs, but not nearly as many as you might think for a company our size.

Our people believe that you don't get anything for free. If they're not paying for it, then they're not going to get anything. And we pay our interns very handsomely. The same with people who are in co-op programs. But first I'd like to seek clarification on a point. Bob, you said that students can't be paid for credit. So if I had a co-op arrangement and I was paying students, they couldn't get credit for what they did?

RB: That's right.

DG: Is that true at other schools? Any comments?

MS: At our institution, I think that it's possible to get credit and get paid. In our department, we would prefer not to do this because there are a lot of problems with this arrangement. The salary is set in a certain way, and the credit is set in another way. Suppose a student got paid and then didn't pass the course. That wouldn't be right.

GC: We don't like it either, but sometimes we bend the rules, and this occurred in a couple of my most successful projects. I do project-based courses in one of which a student, who was a co-op student in the sense that you describe, was breaking the rules by taking the course at the same time he was in the co-op program. His class team was working on the project that was part of his co-op responsibility. Talk about conflict of interest! That's an accident waiting to happen, but that one worked out very well even though there were good reasons why it could have failed.

DG: I'm looking for opportunities and what I am hearing here suggests some. We've had limited experience with student consultancies, and we've not been too enthusiastic about the idea for reasons which are sometimes hard to pin down, but I'm open to new ideas, and I really sense what I'm hearing offers an opportunity to further build a relationship with an academic partner. In industry, we're doing that more and more with one another, and I certainly think that the partnership between education and industry is important.

MS: I think that a lot of the best liaisons that we have had would love to be a college professor. These people come up with a problem, they know it's important, and they love to work with the students. A person who looks at it as a burden is just not going to do things the same way. We have very strict rules about who can contact the liaison. Our liaison lives in his own environment and comes in twice a week to discuss the project. Only the project director can contact the liaison person. We don't want the liaisons to have kids calling them all the time. On the other hand, the best projects come from the people who love being with students. They invite the students to come to the factory, to inspect the production line, to smell all the awful smells, and they all have to wear white caps and paper shoes to go through the factory floor. The students have so much fun doing all this.

MK: Clearly industry liaisons have to believe in the program. They must also provide cover for people who are looking for immediate results. One thing about industry is that it is results-driven, which means that if the job takes three weeks, managers want it done in two. When we start talking semesters, industry becomes very impatient. That's where somebody's got to be there, somebody who can negotiate between you and upper-echelon management which is spending the dollars, to explain why the problem isn't solved yet. There are so many hidden hurdles in industry that can trip you up. The liaison has to be able to steer the students around the mine fields.

The example of the failure that you cited, Dan, where the students came up with the solution that Abbott already knew, probably could have been avoided by better liaison between the students and the company.

DG: Oh, yes! As I thought about that some more, I wondered what it was we were really expecting. But it's easy after the fact to look back and say that it is not going to be a problem how we go from an aqueous solvent to a non-aqueous one. But no student is going to solve that in a short period of time. This is going to be a two-year project, and it's going to require a huge team of people. Maybe somebody thought that a student team could come up with an easier solution, but what happened in this case is that the students confirmed that there is no easy solution. That, in itself, is valuable information.

One other thing: Regulators are volunteering to send their enforcement people into industry sites to do a project. They'll do it, although they want you to pay for it, because they'll do whatever they can to get their people in the door, and that creates some nervousness in industry. Talk about not wanting to open a door! It's just one of those areas where you feel like saying "Whoa, we're not home, we've moved to Arizona."

Regulators are pushing these kinds of projects more and more. We actually have a plant manager who took part in one of these activities, and it turned out to be a non-event. Still I think a lot of people felt that he was climbing out on a limb, and they were afraid of the potential consequences of what he did. But I think that it's a way to educate the enforcement community. There are probably others out there who are starting to take a look at these issues although people are still walking pretty gingerly.

DW: It triggered something in my mind, Dan, when you said that maybe the college folks could come in and see something that hadn't been intuitively obvious. That really rings two bells for me. The first is that some of the best projects for consultancy groups are projects that are not problems within the core business because, as you said, they have to be projects that can be solved over the course of a year. If there is something which is critical today, it's that industry is not patient enough to wait a year for a solution. But, in some cases, there are problems that people don't know are problems, and those are really good consultancy projects. The second thing I heard you say concerns paradigms: if you can give a project to a new set of consultants, they might come up with something that's entirely different. Here's an example of that: the person who developed the modern telephone-switching equipment was an undertaker. Because he didn't know what the paradigm in the telephone-switching business was, he was more likely to come up with a break-through solution, more likely than somebody who knew all the rules. There are a lot of break-through examples like that. And I think that's the real value of fresh blood coming into the organization.

Q: What kind of industry do you think would find interdisciplinary teams more appealing? Clearly a physics group working with a biology group can possess an insight which a single-discipline team might not have. You would think that industry would be more accepting of such arrangements.

DW: I think it depends on the situation. I come from the transportation side of our business, and I'm not in the engineering/manufacturing side *per se*. If you brought me a group of engineers, I'd say, thanks for coming, but you're not going to help me more than a computer person or math problem-solver, or somebody like that. We've had situations where we've brought in graduate students to do project-type work like this. A distribution network design, for instance. It was business, math, and computer-type skills that we were looking for. That work was interdisciplinary in a sense.

MS: My impression here is that maybe we are unable at times to calm the nervousness of some companies that mathematicians are giving them pure answers which cannot be applied in a real world. And so, many times you'll bring in an accounting major or a finance major or a physics major who knows how to hook things up.

MK: From a real-world point of view, interdisciplinary is actually how industry works. It is hard for me to differentiate between a computer science major and a mathematics major and even an engineer because all three of these computer-based technologies are tightly interlocked. In my jurisdiction, we have inspection work and quality initiatives. When you look at those, you're getting into just about every aspect of design all the way through to the supplier base. Our technology changes, just like the technology in education, and everything has become software-driven. So whether it's a math major or not, you're going to be dealing with computers. It's just a matter of *how* you're going to be dealing with them. One other thing that we do at Allen Bradley is to work a lot with a math education professor at Marquette University, in what you would call a math collaborative. We bring teachers from the high school level into Allen Bradley, and at first it was thought that this might not work. But the results these teachers got contributed not only to Allen Bradley but were also used to develop curriculum, and we get the benefit of that as well.

DG: I can't get single-disciplinary projects to work yet. Right now, I don't think we have had enough experience with these although I believe there is value in the concept. I agree with Mark that industry operates in an interdisciplinary fashion. We have people from different backgrounds who, at one time, worked as separate units but are now all meshed together.

GC: Right now I'm teaching a six-week summer class in which I have mostly computer science and math people. I also have some engineers, some students from English, some students from journalism and some students from marketing. They work with a web site that I import, and so the writing and the programming all make it come together. And boy! is that fun. The different students bring different perspectives to the table.

MK: It's interesting how you look at consultancy and how you enter it. One of the things in the forefront of industry right now is the web site. There's been a massive explosion of web sites. I understand Abbott's reluctance to participate because of security issues. But what about internal web sites? We're working with the Milwaukee Public Schools in an intern project whose sole purpose is to bring the web site technology into the Milwaukee public high schools. Now that's a great example of how you start sharing results with those in other areas. We're not really celebrating yet because people in industry are only beginning to appreciate the potential of this new technology, and we don't have everything on the World Wide Web yet. But look at what's going to be happening in the near future!

Course Integration

George Corliss
Marquette University

Contributors to this chapter include Peter Tonellato, a professor of mathematics at Marquette, and Kate McCready, who at the time managed the Funding Information Center at Marquette.

After Bob Borrelli's remarks, you may be saying to yourself, "Gee, I don't have a director, a secretarial staff, and all the wonderful resources Bob has at Harvey Mudd. I guess I can't do anything at all."

False. You and I have the power to decide how we want to teach our courses. I teach classes in mathematical modeling, in computer systems analysis, and one summer in web development using a strong class project. When I walk into the classroom, I can teach my class any way I want. That's one of the wonderful things about this occupation. I can't think of any other job that has the autonomy of a college professor. When I shut the classroom door, whatever happens between my students and me is my business. If I want to run my class in a certain way, I have the power and the authority to do it. If I screw up, the university should hold me accountable, but no one looks over my shoulder.

Over time, I have developed some skills in dealing with groups of students and clients. You may not be able to tell from the color of my hair (blond) that I have some gray hairs, but I do, and it helps to have some war stories to tell. Where do the war stories come from? They come from the projects I do and the students I visit. The more student projects I assign, the more stories I have to tell.

Is there release time or a salary increment for this? Ha! In your dreams. My reward is personal satisfaction. I love what I do. I have the feeling from what I have heard that many other mathematicians do, too. Project-based teaching is very effective. It is very fulfilling to go home at the end of the day and say, "I earned my salary today."

It's very satisfying when a former client calls and says, "You did such a wonderful job for us last time. Do you have another team you can send us?" A good job generates repeat business. I have done some consulting, and sometimes my consulting jobs lead to projects. Sometimes it's the other way round: the projects lead to consulting jobs.

Some of the rewards are the students' commentaries on teaching. If only Marquette would delete the question about whether the workload in the class is reasonable, I would break records. I typically get high scores on the rest of the questions and "0" and "1" on this one. The class *is* a lot of work.

I frequently get messages from students. A few days ago I received one from a student who had worked on a foundry project. He says, "I wonder if you remember me." You bet I do. I remember every one of these students. We sweated blood together. He goes on, "You may recall the group that I was working with back in the spring of 1990. Our client finally completed the installation of that system we conceived six years ago, and what they installed is essentially what we designed."

I get a lot of satisfaction from a message like that. Can you imagine a computer design which is stable six years later? Those students did one hell of a job if the client still wants to install it. It must have been a rather forward-looking design. I only wish I could remember what the design was!

Figure 1. Marquette University, the author's institution and site of the Undergraduate Consultancy Workshop (Courtesy of Marquette Univeristy)

That summarizes the way I try to teach, stuffing course "material" somehow among the projects.

The size of my student teams varies. My experience is a lot like Bob's. Three or four members seems about optimal. Sometimes I do a team with two. Occasionally, I've had teams of six. When that happens, there will usually be a discussion somewhere along the way about the factorial as an example of computational complexity. Large teams don't work as well.

In general, I assign the teams. I know most of the students from other courses, or I talk to faculty colleagues who have had them before. I get the class list in advance, and I put the teams together. However, I don't assign the team leaders. It works better to let the students figure out that they need a leader and define what that role should be. Then they have something invested in the leadership instead of my choosing a leader from equals. In political science, that's called the consent of the governed.

I remember one group of three students who were soldiers. If somebody told them to do it, they'd march into a thicket of cannons, from the wrong end, but somebody had to tell them what to do. They were followers. They're going to be good employees someplace, but not as leaders. Or so I thought.

Another student in the course was a Ph.D. candidate in the department. He was a top-notch graduate student, so I assigned him to the group of soldiers on the assumption that he would evolve into the team leader. Well, he came to class, heard the job description, and dropped the class! It was beautiful to watch the three soldiers who were left. I went to two of the presentations they gave their client. It was as if one person were doing it. They were so tightly coupled as a team that they just seamlessly passed the conversation around the table. One person was talking about something, someone would ask a question, and a different member of the group would step in and answer. I haven't seen one person as coordinated as those three. I really blew that team assignment, but did it turn out well!

A word about class size: thirty is too big. Most of the classes in which I use projects so heavily have 15 to 20 students, usually in three to five teams. My teaching load at Marquette is two courses a semester plus research expectations. I do research in scientific computation. I was overpowered by 30 students in one summer's class, but we got through it, even if it wasn't as much fun.

I do get a lot of help. Peter Tonellato, who will comment on this later, is one of the people who helps me. I also place a lot of responsibility on the students' teams. There are lots of places they can go to ask for help. They can come to me. They can go to their client. They can approach other faculty members. They can go to the library. They can consult with people at other universities. We have students, for example, who visit with faculty members at the University of Wisconsin at Milwaukee. I give the students a lot of responsibility. Many go to people like Peter and say, “Corliss really dumped on us today. Can you bail us out of this mess?”

A 3-credit class has three class meetings a week. My model is that of a consulting company working on various projects. We all sit down in class three times a week, talk about what we’re doing, and discuss the resources that can help. I do whatever I can to move projects along and present some academic content. I typically meet with each individual group for an hour a week besides that.

Peter Tonellato: I want to comment about student presentations. I’ve never heard students complain about that aspect of their work. It is an important aspect of the project, and the students realize that. They’re learning skills that are really valuable, not just math or computer skills, and they genuinely like that. Both George and I have had the experience of former students coming back to say how it improved their skills. “The reason I got my job was because I told the interviewers what projects I had done with Dr. Corliss or with you or with somebody else. When they heard about those projects, they offered me a job that same day, during the interview.”

Of 16 graduating students this past semester, only one is still (late May) looking for a job. Two of the others took their final presentation to an interview. They weren’t asked to do that, but when asked, “What kind of work have you done?” they had the opportunity to show their presentation. Both of them were offered jobs on the spot. I make sure the students know that this kind of experience has great value. From my perspective, companies like what they see. These are the kind of skills that they like, apart from the math communication skills and project development skills.

Corliss: In the last five years, I’ve had three different interviewers call me up after interviewing a student from one of these project classes to say, “I hired a student from your class. My starting offer was five thousand dollars higher than it would have been after they described what they did in your class.” I tell that story on the first day of class.

Let me offer a more detailed description of the nature of the course. It has what we would all agree is legitimate academic content. When I’m teaching mathematical modeling, I have some topics in mind to cover during the semester, including proportion, arguments of scale, dimensional analysis, simulation, elementary differential equations, and stochastic processes. I ensure that we get at least some exposure to those topics. In the systems analysis class, we talk about software development methodology, structured analysis, object-oriented analysis, and rapid prototyping. I make sure that I sneak all these in. Students might think that all they’re doing is the project. I make sure that, by the time they’re done, they’ve had the academic content that we’d all be comfortable with.

The mathematical modeling class is aimed primarily at sophomores, although the class has the reputation of being hard, so many students postpone it until their senior year. The intent of the class is to teach the craft of mathematical modeling, in part as a prototype for computer systems analysis.

Tonellato: The modeling class is required of computer science majors. As George says, they wait until they are seniors to take it because they think that it is a hard class with a lot of mathematics. It got to the point where we were getting many engineers in the class because the engineering students recognized the value of the course. The Department of Mathematics, Statistics and Computer Science decided that it would no longer accept engineers in an effort to limit class sizes. When you get more than twenty students in the class working in four or five project teams, it is too time-consuming to teach.

A comment about our students’ ACT scores: I don’t know them, but our Admissions Office tells us they are good, and I think our students enter with sound skills.

Corliss: I like C students. There are some students at the top of the pile. The best thing we can do is to keep out of their way, not interfere with them, and they’ll do fine. There are some students whom we cannot help, no matter what we do. Then there are the people in the middle. We all have heated discussions in our departments about the majors we want to recruit. Colleagues say, “Get the A students.” I’d rather have C students. There are a lot more of them, and they pay the same tuition as the A students. In fact, the C students may not have scholarships, so they bring more money into the university. Marquette's Development Office is pursuing a former student of ours. He graduated with a 2.05 GPA. I gave him three C's. He recently sold his company for $20 million. By at least one measure of worldly success, he is successful.

The top-notch students don't need project experiences like these. They'll do fine without them. It's the people in the middle who can really benefit from these project courses. We have some pretty ordinary students. One student in my class turned out to be a leader of his team this semester. He had been on academic probation three times. He was in my office yesterday because he's finally been kicked out of the university for doing so badly. Does it matter that he has co-authored a book on Java and otherwise is doing well? I think the university should be happy to help him finish his degree and hope that he'll remember where he came from. He just seems to have trouble with English, theology, and Spanish.

No, the "mediocre" student benefits from the project work we are describing here at least as much as the best student.

Mark Kopczynski: Let me comment on the issue of balancing skills. Here's an example which comes from the way we hire at Allen Bradley. We go through a series of interviews in which applicants are seen by a number of our people in the hiring process. The night before this happens, we have a social hour where everybody can let their hair down and have a few drinks in a friendly setting. One of the people once was an average student, who looked a little geeky. The next day, after I had interviewed him, I gave him a high rating. When the director of human resources asked me why I had done so, I said that the man had brought a paper that he had worked on as a student from Michigan Tech. During our session together, I asked him to explain the work he had done, and what he said showed that he had really benefitted from a process involving interdisciplinary skills. Later that day, when the director of human resources, who had not participated in this session, said that young man would not succeed at Allen Bradley, I replied, "Bring him back and get him to focus on the subject of his paper." We did, and he excelled. He's been with us now for five years, and he's been doing exceptionally well. You cannot always trust your first impressions. Some people who look kind of spacey are pretty sharp when you get them focused and round off the rough edges a little bit.

Corliss: One of our colleagues, Kate McCready, can relate her experience as a client for one of my project teams.

Kate McCready: As a reference librarian at Marquette for the past two years, I am responsible for a collection called the Funding Information Center. We provide information about grants and grant opportunities for the non-profit community. We have traffic of about 1,500 people a year and about the same number of phone calls. In addition to maintaining this collection, we also publish a directory of all the foundations in the state, so all the private funding sources, with details about contact persons and the funder's application cycle, are listed in one place.

This is the Bible of fund-raisers in Wisconsin, but the last thing they want to do is to spend hours going through this book and cross-referencing entries in the index. They want to go to a computer, type *higher education*, have a list of names pop out, and then print them.

When I started two years ago, I was given the job of editing this book, and it's been my goal from the beginning to make this available electronically. After publishing the 1995 edition, I was determined to have the 1996 edition available in both print and electronic formats.

My first idea was to hire an individual. It did not occur to me to hire a group from a class. My first step was to talk to the secretary in the Department of Mathematics, Statistics and Computer Science to see if some faculty member would refer me to a student whom I could hire. I was referred to Dr. Corliss, who said, "Well, you can do that, but you may want to consider working with a group of students from my class."

I got off the phone rather excited at the prospect of having a pool of students instead of just one. I'd have different talents coming from different directions. My supervisor thought it was a good idea, too, so I said, "Great. Let's submit our name as a possible client." I'd used a lot of different electronic products, so I had some idea of what I liked as far as searching capacity was concerned, but I had never done anything about creating software.

One group came to interview me, but they thought it was too much work. A second group thought that it was manageable. There were five students in the group, and we decided to have them create the electronic version of the book and also to create a web site that would advertise the center and the electronic version.

The overall process was much more time-consuming for me than I had anticipated. I thought I would come up with my original design for how the thing would look, and I would give it to them. They'd come back and say, "Here you go," and it would look something like what I wanted. Instead, it was more of a dialogue. I would say, "Here's what I would like." They'd come back and say, "We thought about what you said, and we have some other ideas." I didn't really expect them to come up with these great ideas, and I'd say, "Wow! Yeah, let's go for that." It added to my workload when they brought their own ideas to the project because we had a lot more to discuss, but it turned out to be a great project.

If this type of project is of interest to you, get the word out that you are doing this sort of thing. It was by chance that I happened to contact the Department of Mathematics, Statistics and Computer Science and to discover that there were classes doing projects for individuals and companies.

Our final product is just about done. I hired one of the students to finish up the loose ends of the project. That is another thing that you need to tell your clients: there probably will be loose ends. It's unlikely that you will be handed this neatly wrapped package where everything is all done and ready to go.

The web page is stable now,[1] and we have a down-loadable demo of the electronic version. We plan to sell the product that the students created. In fact, it never occurred to me that I would *not* use what they might develop. That helped the students focus. They realized that I was taking them very seriously. They were my one shot at getting my project off the ground this year. I wasn't going to turn around in the summer and hire somebody else. It was them or nothing, so they were very committed to the project. They put in an incredible number of hours. They'd say, "Well, Kate, how do you want to do it?" I'd say, "Well, what do you think I'm going to say? I'm shooting for the moon. You're going to be the ones who will have to tell me no." Normally they'd say, "If you want it that way, we'll do it that way," and they did. There were very few times when they said, "We don't have time to do this."

Corliss: It might be asked how many of these projects actually end up with something useful. The answer is nearly all of them, in some sense. Many projects in the systems analysis class end up with a design for a system, a conceptual program, a request, a proposal, or something like that. Nearly all of our projects are software related, but many don't get to the point of writing a program.

Kate's director came to the final presentation and said some very complimentary things afterwards. I asked him to share those thoughts with the Vice President for Academic Affairs. We need this kind of news to be shared with upper administration. We need our administrators to know what we can do and that we serve the university. I felt a little strange asking that kind of favor, but it's the good will of people like him that is going to help fund some of our laboratories, even in a very indirect way.

I haven't always had the same experience as Bob Borrelli with on-campus sources. Many of ours have been very successful, like Kate's project, but others have not been so successful because too many university officials thought that they were helping us educate our students, but they were not committed to the consulting process. They might return students' calls a week from Thursday. The best clients are people like Kate who take advantage of my students to get something done. If potential clients think they are doing me a favor, I run the other way.

One semester, we worked with an on-campus office that had been very helpful to one of my students. He wanted to do something for them in return, and they said, "Okay." The project did not go well: We could

[1] www.marquette.edu/library/fic

never find anybody to answer our questions, and when we'd ask a question, they'd just make something up. Some on-campus projects work well, and others are disasters. Help your students interview potential clients wisely.

McCready: From the client's point of view, a semester is a short time. I had no idea what software would have to be used or what development tools would be required. There is a lot of background work that needs to be done before you hire a consultant, whether it's a company or a group of students. This is especially true if you're going to do something you need but don't know much about.

Tonellato: I'd like to mention another projects course. My background is in applied mathematics, and I'm interested in applications which include biology and medicine. I do most of my research at the Medical College of Wisconsin with people who are studying hypertension. We model blood flow and organs. As a result, one course that has evolved into a projects course is called "Mathematical Modeling." It's an upper division course for mathematics, biology, biochemistry, and bioengineering students, usually as juniors or seniors. I used Lee Segel's book on modeling of processes.[2] This is an introductory biomathematics course, and we would just go through Segel. I would teach the standard material: difference equations and differential equations, PDE's incorporating biological data, cellular calculations, and population dynamics. More recently, the course has changed character quite a bit because of the research interest that our department has at the Medical College, the Zablocki Veterans Administration Medical Center (VA), and the Applied Research Institute of Southeast Wisconsin. We find ourselves collaborating at a graduate research level with other medical institutes in the area, and now this course is being integrated into that environment. This is not exactly what we have been dealing with here because it is a customers' institute, but it's the same framework, and our customers are various departments in the Medical College and the VA.

The Department of Biology participates extensively in this work. The mathematical modeling course has expanded somewhat because it is jointly offered between the Medical College of Wisconsin and Marquette University. At the Medical College, a Department of Physiology course in biomathematics attracts mostly graduate students who are doing basic science work in physiology, biophysics, chemistry or biochemistry, and cell biology, together with some M.D. Ph.D's. We also have a group of Marquette students who are mainly upper-division math, chemistry, bioengineering, and biology students. The course is popular because many of the Marquette students want to participate at the Medical College, gaining experience in the lab since perhaps 50% of them want eventually to become M.D's. They are usually highly skilled, they are very motivated, and most of them have had a lot of math. If their mathematics background is deficient, then they compensate by being very bright and knowing a lot of biology.

These are projects funded mostly by the National Institutes of Health (NIH). We start off with a book. Recently it's been *Understanding Nonlinear Dynamics* by Daniel Kaplan and Leon Glass [Springer Verlag, New York, 1995]. We try to inject relatively small doses of mathematics into the biological sciences. It is very easy in these projects to focus solely on computers, databases, or system development, but this is a biomathematics course, and we integrate mathematical content as a foundation.

Even when we attack computer science issues such as system integration, database development, front end analysis, or analysis tools, the core issue is the application of mathematics to biology or medicine. We form groups to balance the skills of the students. A classic situation is to have a senior in biology, a senior in bioengineering, and a junior or senior math major working with a Ph.D. candidate in the Department of Physiology who is in the lab every day grinding out data with little idea how to analyze it beyond taking a mean and standard deviation or a t test.

It's a natural setting for generating projects, although we have to balance skill levels. It's hard to amalgamate the various student backgrounds and work on a single project, but it usually works out well. Those projects also have been seed projects and mini-research projects that drive our research collaboration. The framework is a bit different from the consultancy projects mentioned before, but it certainly has

[2] *Modeling Dynamic Phenomena in Molecular and Cellular Biology*, Cambridge University Press, Cambridge, 1984

Figure 2. The atrium of Cudahy Hall, home of the Marquette Mathematics Department (Courtesy of Marquette University)

benefited my research. There have been computer science spin-offs in database and in front-end design, and a graduate computer science course that I offered last semester generated significant interest from industry.

I've been asked about the math preparation of pre-med students, and I can say a little bit about my experience with them. As a group, they may be self-selected. Most of them major in biology and at least half have taken a year of calculus. Many are interested in medical research. Apart from calculus, we teach a lot of linear algebra on the fly, as we need it. For example, students learn some linear algebra when they do linear stability analysis. I try to stick to the basics. It's very hands-on, very practical. Of course, there are projects that require a more theoretical background. Some of what we do might grow into that, but as George pointed out, our project-based classes are self-contained.

Corliss: In such self-contained courses, the mathematical theory we love is highly motivated and tightly focused. Instead of proving all the theorems I might enjoy, I prove enough to get the answer we need, "Okay, this blood treatment is going to work. Now we'll stop." That approach makes the theory more accessible.

Tonellato: I want to emphasize that we have classes in which we make it very clear that applications are a core issue. The theory should not be ignored, but the application of theory to a particular situation is what drives the course.

This chapter concludes with a comment by Todd Fiorentino, another participant in the Consultancy Workshop, who worked in the 1990's for Arthur Andersen Consulting, a company which, after a period of turbulence in its history, evolved into Accenture.

Todd Fiorentino: I've been at the company for about eight years now. I would add that it's the practical business experience that these students get which is really critical to the consulting business. For those of you who don't know about us, we are involved in management and technology consulting. We have 38,000 employees world-wide, and we plan to hire 4,000 people just in the U.S. and Canada over the next 12 months. Client demand and developments in technology are resulting in more and more application of technology to business problems. These are problems that business needs to have solved to survive in an increasingly competitive environment. Technology is playing a huge role in how organizations need to change and how they need to respond to remain competitive.

Three key things that have been touched on need to be stressed. One is the team approach. We're *always* in a team environment. We balance the skills of the team members to encourage brainstorming, knowledge and idea sharing, and to give everyone the skills they need to develop for the future.

The second key concern is methodology. It's not so much *which* methods the students learn as the fact that they have adopted a structured approach to solving business problems. They know how to do research, they know how to weigh the pros and cons of different solutions, and they know how to make decisions on the basis of such an approach. They know how to be self-starters and how to be critical thinkers.

The third key is technology. As everyone involved in these matters knows, the Internet is playing a more and more important role in business. We have internally something that rivals the Internet, something we call the Knowledge Exchange. It's an internal infrastructure using Lotus Notes technology. Knowledge Exchange is a worldwide infrastructure of databases that we use to share information. If I need to get the right information for my project experts in object-oriented technology, I can go to various technology databases in our organization and find the phone number of a person I need to talk to, possibly to get them involved in my project, to do a presentation, or whatever might be needed. It's our private Internet. It's a way that we can leverage ourselves. Not everyone can be an expert in everything that's happening in the market. It helps us to bring in the right resources to get the balance we need.

It fits right into what our clients demand. The skills that are being developed in these projects courses are exactly the kind of skills we are seeking. The students who participate in these courses are the kind of people our clients demand.

References

[1] Avner Friedman and John Lavery, *How to Start an Industrial Mathematics Program in the University,* SIAM, Philadelphia, 1993.

[2] Daniel Kaplan and Leon Glass, *Understanding Nonlinear Dynamics,* Springer Verlag, New York, 1995.

[3] Lee A. Segel, *Modeling Dynamic Phenomena in Molecular and Cellular Biology,* Cambridge University Press, Cambridge, 1984.

The Consortium for Mathematics and Its Applications (COMAP)

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What's COMAP?

Let me tell you a little bit about the nonprofit organization COMAP and its history. The name is an acronym from the first initials of the words in its full name, and many of COMAP's projects have the syllable "MAP" at the ends of their names. COMAP, the Consortium for Mathematics and Its Applications, was founded more than 25 years ago by Sol Garfunkel, who had been a professor of mathematical logic at Cornell University and a teacher of mine. COMAP has a multi-million dollar annual budget (a substantial fraction of that of the Mathematical Association of America, for instance). Most of COMAP's funding comes from grants, and there are about 20 full-time people on staff at the headquarters near Boston.

It all began with a 1976 NSF grant for the "Undergraduate Mathematics Applications Project" (UMAP) and has continued under its own steam ever since. The COMAP mission is to

- develop new instructional materials and
- create an interdisciplinary consortium within the academic community to produce and disseminate them.

COMAP distributes curriculum materials prepared by college and high-school instructors. COMAP originally started out with print media but has branched out into video, CD-ROMs, and the WorldWideWeb.

The original impetus for COMAP arose from the conviction that at all educational levels, mathematics instruction could be enriched, student motivation enhanced, and student success increased by the introduction of applications—not just realistic applications, but real applications.

UMAP Modules

The original goal was to develop lesson-length instructional units specifically about *applications* of undergraduate mathematics, from a vast variety of fields. A UMAP module begins with two pages of information about the module, including

- the mathematical prerequisites that students should have,
- what area(s) the application comes from,
- an abstract of the application and any mathematics developed in the module, and
- what students ought to be able to do after finishing the module.

The material itself is organized into sections, including exercises and usually a sample exam (with solutions), plus references and a brief biographical sketch of the author.

Figure 1. Sol Garfunkel, founder of COMAP

Sample titles of UMAP modules are:

- Lagrange Multipliers and the Design of Multistage Rockets,
- Glottochronology: An Application of Calculus to Linguistics,
- The Mathematics of Scuba Diving,
- Exponential Models of Legislative Turnover,
- Fire Control and Land Management in the Chaparral,
- Passive Solar Heating,
- The Duration of a Bond,
- Modeling Blood Alcohol Elimination,
- The Mathematics of the Rainbow
- How Does the NFL Rate Passers?

By now, there is a library of hundreds of UMAP modules. Not all are concerned with applications; some deal with basic skills and others with topics in pure mathematics. The modules are available in Adobe Acrobat PDF format on a CD-ROM, together with software to search by mathematical topic, area of application, or courses for which they are appropriate (calculus, precalculus, *etc*.).

The ideal UMAP module starts with a situation in the real world and a particular problem. The situation is then translated into mathematical terms, and a mathematical model is developed. The model uses the mathematics to produce some results. The module then goes on to ask and answer questions such as: What insight does the model give us? Does it tell us anything that we didn't know before? Does it raise further questions about the situation in the real world?

Originally UMAP modules were conceived as units to be completed in one class hour; but once students start getting involved in the ideas and applications, it's hard to get them to stop, so some modules may take up to three class hours. The modules are designed to be incorporated in regular courses, although there have been entire courses constructed entirely out of just UMAP Modules, particularly courses in mathematical modeling, as was one textbook for the subject [Giordano *et al.* 2003].

The UMAP Journal

All government money comes to an end eventually; a few years after the UMAP project started, Sol Garfunkel founded an organizational entity, COMAP, to carry on the work. In addition to distributing individual modules (in print form then), COMAP's Board of Directors decided to start a quarterly journal featuring them and other materials related to *mathematical modeling and applications*. As a result, *The UMAP Journal (of Undergraduate Mathematics and Its Applications)* began in 1980. An issue usually contains one or more new modules in their entirety. In addition, there is an editorial—often, a guest editorial—advocating a particular point of view. There are articles, too, plus book and software reviews. As time has gone by, the *Journal* has evolved, so that now it includes Minimodules, ILAP Modules, and papers from the Mathematical Contest in Modeling (for more about these, read on).

In addition to the Modeling Contest papers, students have published other articles and modules in the *Journal*, some with faculty co-authors and others on their own. One student analyzed the arbitration between labor and management in a firm in Rockford, Illinois. Another student co-authored a minimodule about backgammon with Arthur Benjamin from Harvey Mudd College, who has done several articles with students. Paul Isihara and students at Wheaton College have a recent module about the spread of HIV/AIDS.

In the "On Jargon" department of the *Journal*, there is room for articles that are not immediately related to applied mathematics or to modeling. This occasional column usually explains one particular mathematical idea, concept, or term and gives the reader some insight into how it fits into mathematics.

People who feel strongly about ideas related to the teaching of applications of mathematics are welcome to submit potential guest editorials. We don't get an overwhelming volume of manuscripts, as *Mathematics Magazine* does (300/year); consequently, we're able to publish much more of what we get. Perhaps half of what appears in the *Journal* is by invitation: I or an associate editor hears a talk, for example, and we ask the speaker to submit a paper to the *Journal*[1]. But it also is a wonderful Christmas-like surprise for me to find in the mail a packet from someone whom I've never heard from before, with an exciting manuscript inside.

The **Tools for Teaching** Volumes

Only some UMAP modules appear in the *Journal*; there isn't room for all of them, particularly the longer ones, so COMAP also publishes an annual book supplement to the *Journal*, containing additional UMAP Modules and other materials. Modules that appeared in the *Journal* in the course of the year appear again in this book. You may ask why we publish them twice. The answer has to do with how libraries buy items and shelve them. A library may not have a budget to subscribe to a journal on a continuing basis, but it can buy a volume a year in what librarians dub a *serial*. Such acquisitions tend to be bought out of regular book budgets rather than from money allocated to periodicals. We feel that we are providing a way for libraries and mathematical sciences departments to acquire a compendium of resources.

A number of professional organizations are invited to nominate members to the *Journal*'s editorial board, to help provide a pool of qualified individuals for refereeing submitted materials:

- the Institute for Operations Research and the Management Sciences (INFORMS),
- the Society for Industrial and Applied Mathematics (SIAM),
- the Mathematical Association of America (MAA),
- the National Council of Teachers of Mathematics (NCTM),
- the American Statistical Association (ASA), and
- American Mathematical Association of Two-Year Colleges (AMATYC).

[1]See the chapter on ICIC projects for an example.

A paper submitted to the *Journal* is usually sent to at least two referees, including a member of the editorial board. Over the 20 years that I have been the editor of the *Journal*, I have been very grateful to the members of the editorial board. Some have been authors themselves and others have kept their eyes out for good contributions, going out to hear talks and asking, “Why don’t you write that up and send it to *The UMAP Journal*?”

Projects at the High School Level

After establishing COMAP, Sol Garfunkel went on to secure other grants. With the college-level enterprise continuing under the UMAP umbrella, COMAP branched out in 1984 to the high school level, producing “HiMAP” modules. The main difference, apart from the fact that HiMAP modules are designed to enhance the high school curriculum with applications, is that HiMAP modules include black masters (from which a teacher can make transparencies for class use) and worksheets. Thus, HiMAP modules are oriented more closely to work that takes place in class, whereas UMAP modules involve a more independent approach in which students do most of the reading on their own, perhaps with some class discussion and some lecturing by the professor.

Just as COMAP founded a journal to continue UMAP activities beyond the original grant-funded period, COMAP also instituted *Consortium*, a quarterly 16-page, high-school-focused tabloid newsletter. It goes to 50,000 high-school teachers, more than half the membership of the National Council of Teachers of Mathematics (NCTM). This newsletter has sections on technology in the classroom, historical notes, and classroom activities, as well as on COMAP’s High School Contest in Mathematical Modeling.

More recently, in its ARISE project, COMAP undertook a major initiative to prepare a comprehensive high-school mathematics curriculum and accompanying textbooks [COMAP 1998–2000]. The first volume, for 9th grade, came out in 1998; half a dozen years later, we are in the midst of the cycle of preparing the second editions of the four volumes. Again, the focus is on applications, but there is a major difference from other high school texts, which tend to tack on a few applications to make the mathematics look relevant. The prevailing attitude in these other book series is that there is a mainstream core curriculum that has to be covered and the applications are there as an appendage, sometimes as a window-dressing sidebar, to be covered if time and the teacher and student interests permit. The COMAP series *makes mathematical modeling the fundamental activity and purpose* of the courses. To pursue the modeling brings into play all the other concepts and skills that the mainstream curriculum emphasizes.

The Mathematical Contest in Modeling

Another milestone in the history of COMAP occurred in 1985, when Ben Fusaro of Salisbury State University in Maryland had an idea. Since there is the annual William Lowell Putnam Contest in pure mathematics for undergraduates, he thought there ought to be an “applied Putnam,” as he called it, in applied mathematics for undergraduates. He suggested to Sol Garfunkel that given COMAP’s distinctive orientation toward applied mathematics at the undergraduate level, COMAP was the natural organization to sponsor such a contest. Based on his idea, with a little NSF funding, COMAP established the first Mathematical Contest in Modeling (MCM) in 1985. Of course, the funding eventually ran out, but the contest has continued and grown in participation ever since, thanks to support from other organizations. It has also broadened to include a branch designed to involve students from other disciplines, the Interdisciplinary Contest in Modeling.

The Mathematical Contest in Modeling (MCM) takes place over a weekend in February. Participating students, in teams of two or three, are presented with a choice among two or more open-ended problems, like one from the 1995 contest which posited that the world’s oceans contain an ambient noise field:

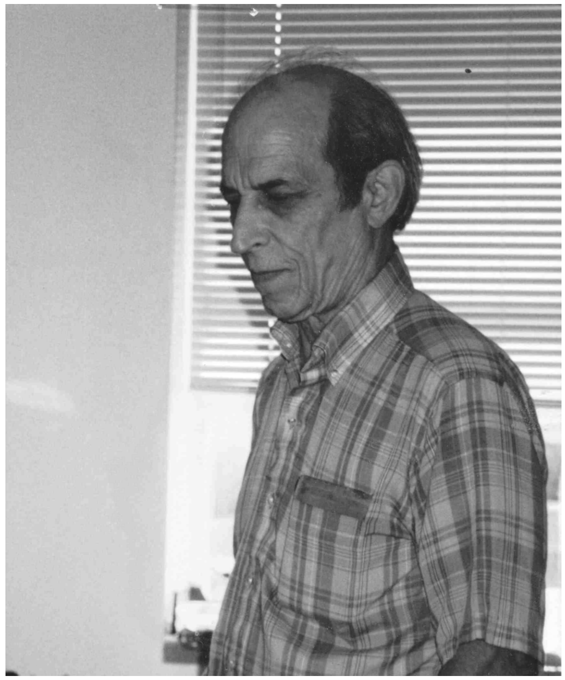

Figure 2. Ben Fusaro, who originated the MCM

seismic disturbances, surface shifting, and marine mammals all contribute to this field, and they do so with different frequencies. The question in the 1995 MCM asked students to consider how this ambient noise might be used to detect a large object such as a submarine, moving below the surface. Participants were told that they could assume that the submarine itself would be making no intrinsic detectable noise. (We're not beyond allowing participants to make some big assumptions, right off the bat.) They were to develop a method for detecting the presence of a moving submarine, its speed, its size, and its direction of travel, using only information obtained by measuring changes in the ambient noise field. Instructions advised participants to begin with noise at one fixed frequency and amplitude.

MCM participants not only work very hard but also occasionally come up with surprisingly ingenious solutions. Without a lot of thinking, I wouldn't know where to begin on this problem, and many participants didn't, either. About a third of the student teams elected to do this problem; the others chose an alternative problem. Usually participants are given a choice between one problem that involves continuous mathematics and another that is likely to involve discrete mathematics. The alternative problem that year was to model a very difficult problem that COMAP itself faces: judging the MCM itself, that is, figuring out a strategy to judge the papers.

Participants can download the problem statements from COMAP's website at any time after the beginning of the contest on Thursday and have until a specified hour on the following Monday to prepare a solution paper. In addition to the body of the paper itself, with any needed figures and tables, a contest entry includes a summary plus additional documentation, e.g., source code of computer programs written by the team.

In 2004, a total of 638 teams participated. They were drawn from 245 institutions in the United States and 9 foreign countries.

At Beloit and at many other institutions, each team is given the exclusive use of a room for the duration of the contest, is equipped with at least two computers loaded with assorted software that may prove useful, and is provided access to a printer and a copier. Many schools provide a small budget for food as well. Beloit's participants tend to live in the room during the contest. By the end of the contest on Monday, we need a special visit from the janitor so that the room can be used as a classroom again on the following day.

The contest announcement is sent to the chairs of all the mathematics departments at four-year colleges and universities in the United States, although many teams are entered from other departments, too, such as systems analysis, industrial engineering, and physics. For many years, several teams entered each year from the Northwest Technical Institute in Wausau, Wisconsin, where Robert J. Henning had great success in attracting participants (many of whom worked full-time), as well as faculty from a variety of departments to serve as mentors and coaches.

Financial support for the MCM, initially provided by the NSF, now comes from the National Security Agency (NSA), COMAP, and an entry fee charged for each team. The U.S. Military Academy at West Point and Harvey Mudd College host judging sessions. Additional support, in the form of judges and prizes for outstanding teams, is provided by the Institute for Operations Research and the Management Sciences (INFORMS), the Society for Industrial and Applied Mathematics (SIAM), and the Mathematical Association of America (MAA).

Judging

To blind judges to the identities of a team's participants and their institution, no identifying information appears on the contest entry itself, which instead bears on each page only an assigned registration number. Initial judging for each problem is done by a triage group of judges that pares down the number of papers, based on their summaries. In March, there is a gathering at Harvey Mudd College of 25 judges who read the papers that survive the triage. There the judging proceeds in a series of stages; at each stage, two additional judges devote an increased amount of time to reading each paper that has survived winnowing. Eventually, the papers are divided into categories, beginning with "Successful Participant," which means that the team made a substantial contribution to the problem: they didn't just totally blow it off, write three lines, and sign their names. Above that category, there are the categories of Honorable Mention, Meritorious, and Outstanding. The proportions in each category vary each year with the quality of entries. In 2004, of the papers which fell in the category of Successful Participant or above, 50% were rated Successful Participant, 33% Honorable Mention, 15% Meritorious, and 2% Outstanding.

The UMAP Journal Publishes the Outstanding Papers

The problem statements and all of the Outstanding papers are published in *The UMAP Journal*, together with a report from the Contest Director and a listing of all participating institutions and the names of the teams' faculty advisors. The 1994 volume in the *Tools for Teaching* series includes each of the 20 problems and one solution paper per year from the first 10 years of the MCM [Campbell 1995].

In 2004, there were 3 problems and 16 Outstanding papers, spread over two issues of the *Journal*. The papers appear in print largely as written, but with some editing by me for style and particularly for length. I don't include the computer programs, I have to omit about half the graphs and tables, and I try to pare down and condense the prose somewhat. But other than that, it is the students' own words and their modeling as they express it.

Along with the papers, we publish one or more commentaries for each problem, including one from a judge for the problem and another from a professional in the field of application from which the problem came. Both commentators make remarks about the problem itself and its context, as well as on specifics of the solutions in the Outstanding papers. To ensure impartiality, they do not know from what institution a given paper comes.

The Contest Director's report tries to put the participants' efforts in perspective, in particular to make the readers realize that each paper is the effort of three students over just one weekend—not a semester-long project, or year-long project by a team of professionals—but an endeavor on which the students exerted a lot of thinking over a short period of time, during which they were not allowed to talk to people outside the team, nor did they have a chance for subsequent revision. They weren't allowed, for example, to go ask their faculty advisor, "How should I do this?" Participants are allowed to consult inanimate resources such as libraries but are forbidden to interact with anyone who has professional expertise on the subject at issue.

I think that most faculty who have had students in the MCM would agree that even if a team's paper does not earn the coveted designation "Outstanding," the team members derive a significant educational benefit from participation in the MCM, comparable in some degree to a semester-long project, with the additional advantage of having done the project as part of a team. Under the banner HiMCM, COMAP now offers an annual mathematical modeling contest at the high school level.

Interdisciplinary Lively Application Projects

Interdisciplinary Lively Application Projects (ILAPs) are small interdisciplinary group projects designed to motivate mathematical concepts and skills. They are *problem-solving projects* and in that respect also differ from UMAP modules, which teach content through exposition, examples, and exercises.

An ILAP contains:

- background information and introduction to the situation,
- project handouts,
- requirements to be fulfilled in a solution, and
- other supporting materials (sample solutions, further background material, notes for the instructor) as appropriate.

Like case studies, these projects often require students to use scientific and quantitative reasoning, mathematical modeling, symbolic manipulation, and computation to solve problems, analyze scenarios, understand issues, and answer questions. The level of prerequisite skills varies; the final product is an oral presentation or a written report.

An ILAP project provides a broad interdisciplinary experience. Hence it must have at least two author-collaborators, preferably from different disciplines. ILAPs have appeared in the *Journal* since 2001. Additional ILAPs appear in Arney [1997].

For All Practical Purposes

What many students fear about mathematics when they go to college is that they're going to be required to take a mathematics course in which they will see the same old algebra that they had before (and in which they may have done poorly). They learned it before, after a fashion; but they always knew that it would come round again, so they didn't really bother to learn it very well.

Here's what George Corliss would call a war story. Some years ago, I was teaching first-semester calculus. A freshman came to me two or three weeks into the class and said "You know, there's something very wrong with this course." You've got to listen to someone who says that, so I said, "OK, why don't you come to the office and tell me about it." When we got there, I said "You feel something is wrong with this course. What do you think it is?" He said, "We're not supposed to do anything new until Christmas."

Two realizations dawned, one for each of us:

- I realized what may go on in some high schools today, at least from the student's point of view; and
- I told him, "This is college."

I hope that that realization helped the man adjust to the changed situation and get through the rest of the term. We weren't going to repeat what he'd learned before. Students may be comfortable with repetition but they also get sick of it. They especially don't like repeating mechanical tasks, e.g., 40 exercises applying the quadratic formula. They can do those things if they have to, but they won't do them until they see some motivation beyond learning the mechanics.

The guiding idea behind what happened next for COMAP was the startling realization by Sol Garfunkel and others that *the average student takes just one mathematics course in college*. Mathematics departments have just one chance to

- give students an appreciation of the place of mathematics in society, particularly contemporary mathematics;
- allay their math anxiety;
- convince them that mathematics is something that they—and their children—can do.

The result in 1988 was the textbook *For All Practical Purposes*. Now in its 6th edition [Garfunkel *et al.* 2003], it has also appeared in German, Greek, and Spanish. The book has units on:

- management science, dealing with planning and scheduling, showing how graph theory, algorithms, heuristics, and linear programming are used;
- statistics, including producing data, describing data graphically and numerically, calculating probability, and reaching conclusions from statistics;
- the digital revolution, including the use of error-correcting codes (such as those UPC codes that students see on everything these days), cryptography and data compression, and the logic of making queries on the WorldWideWeb;
- social choice and decision-making, involving the surprising mathematics of voting methods, dividing things fairly, apportioning legislatures, game theory and competition, and whether there may be a better way to elect a president;
- size and shape, examining how objects and living beings scale as they grow in size, the various kinds of symmetry in nature and art, and patterns to fill space: tilings, quasicrystals and the art of M.C. Escher;
- models for consumer finance, including saving, borrowing, and the relationship between economics and ecology.

There are 23 chapters, far too much to cover in a single course. The book is designed to be used for a variety of courses, for which the instructor can pick and choose which of three or so units to cover. The attractive format, the contemporary applications, and the fact that *almost all the mathematics is nothing like what students have seen before* make this book useful and exciting. Through its many editions, it has been used by more than half a million students.

There is also an accompanying video series, 26 half-hour videos made in connection with the first edition of the book. Some of the examples in the videos may appear dated, but the content is still quite relevant.

Other COMAP Endeavors

COMAP has branched out in an increasing variety of projects at all levels of education.

- For some years COMAP had a newsletter, *The Elementary Mathematician*, for teachers in kindergarten through 8th grade.

- Building on the success of the video series accompanying *For All Practical Purposes*, COMAP did a video series of 26 half-hour programs in statistics, *Against All Odds: Inside Statistics*. The content and sequencing is integrated with Moore and McCabe's statistics text [2003], probably the most successful introductory statistics text ever published (over a million copies in four editions).
- COMAP's video series, *In Simplest Terms: College Algebra* shows students that what they are learning in college algebra is not just "pure" mathematics but mathematics which can be—and is—applied. Pure mathematics has its own beauty, of course, but some students find the subject more appealing if it comes with or arises from applications; and even for those who do find pure math appealing, it enriches perception of the world around us to know, for example, that conic sections are used in everyday objects such as automobile headlamps and telescopes. COMAP has repackaged videos in this and the other video series into smaller, more usable segments, particularly for use in high schools. It's nice to show a half-hour video, but that pretty much kills the entire period. So five- and ten-minute clips turn out to be a useful way for teachers to work topics into class time more conveniently.
- In the 1980's, computer scientist Tony Ralston (retired from SUNY Buffalo) asked, "Is calculus the right thing for people to take at the start of their college careers?" Ralston's interest was particularly computer science. He favored discrete mathematics and made a case for it, at least for students potentially interested in computer science.

  Glen Brookshear in the Computer Science Department at Marquette University has written a very successful introduction to computer science [2002]. It doesn't teach programming, but it exposes students to a wealth of ideas and explains what is involved in the field and what they can expect to find in later courses. A course built around Brookshear's book can be viewed as a kind of mainstream introduction to computer science—not just for poets, but for most students.

  By comparison, studying calculus—not just for a semester, but for as much as the first year and a half of college—doesn't always give students much sense of what mathematics is about.

  COMAP, too, has tried to suggest an alternative to the mainstream Calculus I and II courses of the first year of college, in the textbook *Principles and Practice in Mathematics* [Meyer *et al.* 1996]. It attempts to realize some of Ralston's ideas in a course suitable for students in many disciplines while still opening the door to the study of calculus. The first chapter, for example, treats change, which of course leads into calculus; but the approach is discrete, through difference equations rather than continuous mathematics and differential equations. Students who take a course from this book will be well prepared for the future, whether they go into a regular calculus course or not. Quite possibly, those going on to calculus will have enough maturity to cut the traditional year of calculus back to just a single semester, in which all of the important ideas and techniques are presented.

  But this model from COMAP has not caught on, any more than Ralston's ideas have. Partly in reaction to his challenge, partly because so many students do poorly in calculus, and partly as a response to widespread computer and calculator technology, the calculus reform movement arose and flourished for a dozen years. The result of that movement was to keep calculus entrenched as the standard first-year mathematics course, although it has evolved into a different course than the one of 20 or more years ago.
- COMAP has offered training institutes for college teachers of mathematics, centered on particular topics, especially those treated in *For All Practical Purposes*. One year, the workshop focused on geometry. Faculty members who attended worked as teams to produce modules by the end of the session.
- DevMAP, the Developmental Mathematics and Its Applications Project, has the goal of creating a new two-semester undergraduate developmental mathematics sequence that embodies the philosophy of reform in the spirit of the NCTM Mathematics Standards. The course
  - assumes a graphing calculator, spreadsheets, and geometric utility programs;

   - presents mathematical ideas as growing out of the context of contemporary applications; and
   - encourages group activities and a more open pedagogical approach.

- TechMAP's goal is to create a series of modules for use in high school mathematics courses that feature more technically oriented contexts, such as manufacturing, agriculture, finance, and so on. These units are intended *for all students* rather than for those tracked into a vocational course of study. Far too often, important and interesting technical applications are considered inappropriate for mainstream core curricula—but we all need to learn how the world works and benefit from doing so.
- New publications include activity-based and modeling-driven textbooks for college algebra and precalculus [COMAP 2001a; 2001b].
- COMAP's MathServe "contest" is a way to promote the idea that mathematics can be done for public good. The goal is to build bridges between mathematics departments and community service providers. In one direction, such bridges carry discipline-based public service to communities; in the other, bridges take mathematics students to active learning of mathematical modeling, applied mathematics, and the practice of consulting. In both directions, these bridges convey to students and to community members the power of mathematics to serve the community.

  MathServe is not a competition. Rather, we endeavor to recognize mathematics departments for doing good works and thereby encourage other mathematics departments to do likewise. Each year, we publicize in the *Journal* projects that make a difference—not because the mathematics is new or because the application is unique—but because the projects embody an effective partnership.

  Previous Outstanding MathServe projects have involved:

   - aiding the restoration of the California condor,
   - testing the improvement of inner-city students in a mentoring program,
   - formulating an inventory model for a major industry in a small town,
   - improving the routing of a special bus service for the disabled,
   - contributing to improved effectiveness of an emergency medical service, and
   - evaluating the impact of various proposed solutions to a traffic bottleneck.

The Future

How many of each year's 1,100 new Ph.D.'s in mathematics know about COMAP's efforts to broaden awareness of applications in teaching mathematics? We may soon find that some have even been students who used *Mathematics: Modeling Our World* as high school students.

What we really want is for people to test and use COMAP materials. We need to know how they go. This kind of communication is easier now in the age of the Web, when people in a list group can write things like, "Well, I used such and such a UMAP Module, and I would rather like to see different sorts of exercises or test questions, like the following. . . ." There's still plenty of room for change and development, and we look forward to it.

In response to a question about coaching a modeling team, Campbell referred to essays by coaches in the volume about the first ten years of the contest [Campbell 1995], including those who had coached Outstanding teams. Robert Fraga, who coached a number of teams at Ripon College, one of which obtained an Outstanding result, offered the following observations:

Fraga: The first year I coached a team, I simply approached three of my students to ask if they would do it. They were willing, and they did very well. What I have done at Ripon is to give students questions similar to those found in the MCM. If it is at all possible, I require them to write up whatever solution they want to submit. That gets them in the habit not only of working as a team, but doing the writing.

In 1996–97, I worked at LSU, and I had a team work on the submarine problem Paul was talking about earlier, and I think they did pretty credible work, but not unlike other teams, they thought they had the problem absolutely licked on Friday night. They discovered on Sunday they had no idea what they were doing, started all over again and did the write-up at about 1:00 in the afternoon on Monday, so what they submitted to MCM was two pages long with extensive code. I think it is a very valuable exercise to get students practicing on these questions. I have a stockpile of questions that colleagues have given me over the years. I've also collected some from sources outside the college. I think that's good preparation for the contest, and I also think that this is an excellent educational experience. Students get the sense of the team dynamic as well. I have organized MCM preparation in the context of a club. I try to include a social dimension. This means providing refreshments for the students. They enjoy that and it keeps them coming back for more.

Campbell: Let me say something about the source of MCM questions. It varies a great deal. Yves Nievergelt of Eastern Washington University has contributed several questions and commentaries over the years. One of our problems a few years ago had to do with an idea that he got from visiting a state prison in Washington State, where a question arose about composting the waste from the prison kitchen. The problem that appeared in the MCM was abstracted from that. Some questions have included specific data. In one, for example, the floor of a pond had to be approximated from given grid points. A question about heat loss in a concrete slab was derived from an unpublished problem by Murray Klamkin. Sometimes problems come from real companies with real applications. Sometimes the company is identified and sometimes it isn't. Most of the time, problems are changed enough to make them tractable. They also have to be formulated in a page or two, and for that you have to leave to students' imagination many of the details or else tell them to neglect those details. For example, they were told to neglect the sound of the submarine in the ambient noise problem. I once submitted a problem about a warehouse: A variety of rolls of cloth are used to cover furniture, and there may be many rolls of the same cloth. An order comes in and you need to decide which roll to use to fill the order; you don't want to have a lot of rolls left over with very little cloth on them. This is a cutting stock problem; it was a runner-up to the MCM problem that was used that year.

Some of the problems come out of operations research. Search and destroy missions lie at the origin of some questions like one which involved drug smuggling. That was adapted from a classified problem. There has been a great deal of interest in mathematical modeling and great usage of UMAP and ILAP Modules at the U.S. Military Academy, the U.S. Coast Guard Academy, and the U.S. Naval Academy. The U.S. Air Force Academy and the the U.S. Military Academy have had several Outstanding teams in the MCM.

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Paul Campbell is Professor of Mathematics and Computer Science at a liberal arts college in southern Wisconsin with a student body of approximately 1200. He served there as Director of Academic Computing from 1987 to 1990. In his article, however, Campbell writes about an organization which provides students with excellent background material and helps to prepare them to experience successfully a consulting/clinic type course. For those mathematics teachers who are unable, for one reason or another, to develop a consulting/clinic course, projects such as those found in UMAP modules can be assigned students to give them the flavor of a consultancy project. Campbell has been the editor of The UMAP Journal since 1984. An allusion to COMAP publications is made in the Borrelli chapter on source of problems; another mention occurs in the final chapter.

Program Management

Martha Siegel
Towson University

I come from a different type of institution than other contributors to this book, and I want to point out that it is possible to do student consultancy projects without having the kind of institutional support that Bob Borrelli has. This chapter tells a little about the history of our projects and the way we manage the program. I also will give you some ideas on alternative management schemes.

In 1979, a small group of mathematicians at Towson decided that Claremont had a very good idea. We invited Jerry Spanier, then the Head of the Claremont Clinic, to come to Baltimore. Our institution is a public, state-supported, metropolitan, comprehensive university. We now have about 18,000 students, about 30% of whom live in dormitories or in apartments near campus. The average age of our students is about 25, but the variation is quite large. So we do have a fair number of non-traditional students. In the Mathematics Department we have several major tracks including applied mathematics, pure mathematics, secondary education, actuarial science, and double majors with computer science and with economics. There is a separate (and now rather large) Department of Computer and Information Sciences.

Towson University NSF Grant

In 1980, we received a grant from the National Science Foundation to begin our Applied Mathematics Laboratory. At the time we were the only school trying this type of student consultancy program with teams comprising only undergraduates. One of the reasons we started the program was to benefit our students who were leaving with degrees in pure mathematics but going into applied and industrial jobs. But another important motivation was to educate our faculty! Most of the department was tenured and had been trained in pure mathematics, but very few of our students were choosing that track in the major. Our students were at that time mostly first-generation college students. Their parents were sending them to school or they were paying their own tuition so they could get good jobs upon graduation. They did not see graduate school as a realistic option and they wanted to see the “practical” in their studies of mathematics.

We all know that to do good mathematics, you do not have to do pure mathematics. Pure versus applied: there is good mathematics in both. And people have to study quite a bit of pure mathematics to have the tools to do good applied mathematics. But the students asked to see the applicability of what they were studying, and the faculty, quite frankly, had no idea what to tell them. We viewed the NSF support for our Applied Mathematics Laboratory (AML) as a chance for both faculty and students to learn about the mathematics of the workplace.

This presented a problem. I had to be persuasive to induce faculty to lead a project. I found it relatively easy to be the Director. My role was to solicit projects by finding sponsors. I supervised the progress of the student teams and kept track of deadlines and communication between the project leaders and the sponsors’ liaisons. But the faculty member who agrees to take on the project leadership role can be very nervous about taking on such a responsibility, even if he/she is a very good mathematician.

Faculty Participation and Support

The first thing we did in proposing our AML was to agree that each team would have two faculty leaders. This is definitely more expensive than the Harvey Mudd model, and if you have more experienced applied mathematicians or graduate students who can co-lead a project, you could get by with one leader. In our relatively new Master's program in Applied and Industrial Mathematics, all students must do an internship, and one option is to co-direct an undergraduate student team project for the AML; however, at the start of a program like this, I would suggest that you try to have two faculty members share the responsibility until they have had some experience.

Let me say something about the people who are willing to undertake these projects. To start, we offer them something for doing it. The money comes in the form of summer support for research. We traditionally give $4000 to each faculty member in a summer stipend and pretty well insist that they limit the amount of teaching in summer in order to allow themselves the time to do the preliminary research necessary to begin the project when the fall semester begins. For some faculty members, that is enticing. Furthermore, we generally allow no more than six students on a team. Having the opportunity to work closely with a small group of very good students appeals to our faculty. Each semester, we give 6 hours of teaching credit to one of the faculty members and give the other faculty member 3 hours teaching credit although the course itself only generates three credit hours for one person. This means that each semester, we are obligated to repay the department for 6 hours of release time for each project. The department is willing to take replacement costs, which amount to $700 per credit hour.

Keep in mind that this was during the era when the teaching load was 12 hours a semester. The situation at Towson has changed somewhat of late. But my point is that it is possible to run this program with a limited amount of support: $\$4000 \times 2 = \$8000$ for the summer, and $4200 each semester, for a total of $16,400 per year. As director, I got a 3-hour reduced course load for the spring semester that cost us another $2100. While this is too expensive for a department to fund on its own, we believe strongly that sponsors should be asked to fund research.

In general, both faculty members are mathematicians, but on occasion we have assigned a co-leader from another department. There are situations in which we need the leadership of someone in another science or in management. This requires careful negotiation with their home department.

Our faculty benefit from the projects in ways that are both intended and unexpected. They learn a little about the world outside academe and they learn ways in which mathematics is applied in the industrial or business setting. They bring this back to the classroom. In addition, they form close relationships with the students on the team and have the chance to work closely with good students in mathematics as well as in other disciplines. Although few at Towson have taken advantage of the opportunity, faculty can publish a paper on their research after proper consultation with the sponsor.

Sponsor Fees

The first project we did under the NSF grant was done with the university as the sponsor. It seemed prudent to see if with two faculty members and a team of undergraduates we could accomplish enough to go out into the community and solicit paying customers. Our project for the university was extremely successful. We were ready to go out and charge for our services. We started by charging $15,000 and eventually charged a little more than $20,000 for a year's work.

We tell our sponsors that they are getting a fully researched project with interim and final reports, both written and oral. They frequently get software and documentation as well. We feel confident that in all of the projects we have completed, the fee was justly earned. We gave a great product!

Cooperation with another institution that is nearby or accessible by internet or interactive video can make the expenses more affordable by each and perhaps provide a subsidy to whatever the sponsor can pay.

Students and Benefits

We think of our AML as a capstone experience for those of our majors who are involved. For us, this is a way of bringing together lots of ideas. We expect to integrate a variety of mathematical tools and principles that we have tried to teach our students. Clearly the applications are real. We spend a lot of time on the written and oral presentations to the sponsor. We find that though students feel at the outset that they are preparing for an immediate career path, in fact many of our students begin to think seriously about graduate school as they realize how much more they need to know about mathematics (pure and applied) to embark on those careers.

We do not restrict this course to our majors, however. We choose the team to fit the project and frequently use students from other majors to round out the skills we need to bring to the problem. Most often we have computer science majors join the mathematics majors, but sometimes we have had teams in which half the students are accounting majors or physics majors. There is a healthy working relationship among the students no matter what the major.

Generally, the sponsor liaisons love working with students. They think that being a professor is the best job in the world! The liaisons make a real attempt to get to know the students and to encourage and teach them as we go along. Students realize that the sponsor is putting money into the project and they respond very well to the responsibility. They learn to work in multidisciplinary teams and they learn to meet deadlines. Sometimes students do so well on a complicated industrial problem that the sponsor tenders a job offer to keep one or more of the students involved in the project after the academic year. This has happened in several instances.

Facilities and Equipment

We felt strongly from the beginning that the AML should have its own room at the university. We believed that students on the team should have access and be able to work there at all times of day and night. In addition, the room should be secure since the AML frequently deals with confidential and proprietary information like company data. Although it took persistence, we did secure a small room with two windows, the best computing equipment, a good printer, bookshelves, a seminar table, and a lock on the door. This may seem like a small thing, but space is one of the most expensive and valued commodities on any campus. This was a coup, took a lot of nagging, and it has meant a lot to the students. They see this as their enclave and they are very proud that they are part of the AML. These little pluses attract students and give the lab a kind of honors aura.

We also need good computing equipment. Our budget has a small amount built in for equipment and software. We manage to replace equipment about once every three years and the software we use is frequently obtained from the sponsor. I urge you to keep building funds for replacement equipment and up-to-date software. The sponsor should be responsible for supplying idiosyncratic software.

Outreach and the University

There has been an unexpected fallout from the partnership we built with the community. People for whom we have done projects became our friends and our champions. Towson University suffers from being the second largest of the public institutions in the State. The flagship, at College Park, is the largest and most prestigious. The assumption is that Towson is still the old Teachers College from 100 years ago. We used to feel like a stepchild. And though we began life as a normal school and eventually became a state teachers' college, we are now a large metropolitan university with students in many professional areas other than education, including mathematics and computer science. What became interesting was that although our sponsors may have thought of us as a small part of a teachers' college when they began to talk with us, they slowly began to see us as having a lot to offer them in business, scientific, technical, and mathematical expertise.

Our first sponsor, Westinghouse Electric Corporation, became our first champion and before very long, the company liaison was helping us form an advisory board of business and industrial leaders in the Baltimore region. Eventually, what began as the AML Advisory Board grew into the Industrial Academic Partnership of the College of Science and Mathematics, with representatives from 25 or 30 business, government, or industrial enterprises. This group sponsored lectures, found ways to make equipment available for faculty and student use, and continued to help us find undergraduate research projects and internships for our students.

Do not underestimate what you can accomplish by reaching out. At one point, a Maryland State fiscal emergency led to a suggestion that we drop chemistry and physics as majors. Our community-based group went into overdrive and protested to their friends in the legislature and in other businesses. They were our biggest supporters and we avoided the cuts.

It is important to cultivate a relationship with your university's public relations people. You want your research projects to get press coverage. Some companies are happy to lend their public relations staff to reinforce yours to get their good deed (sponsorship of student research) into the local paper. It is the best advertising you can hope for. Companies will also be willing to use their house publications to show your students working with their employees.

Solicitation of Projects

Many people have asked me how we solicit projects. I think you need a reasonably good-looking brochure that explains what your operation is all about. This can be developed with the university public relations staff, though we found that they were not particularly good at the narrative piece. Most of the solicitation should take place in the fall and early spring. I sent out about 60 letters every February. To build up a mailing list, you might look to your liaison groups on campus. I used the Industrial Academic Partnership and my own personal links to people in the community. In addition, you might get good results from your university's Development Office which may know business executives and government leaders who have the contacts you need. I kept an up-to-date list of all members of the Maryland Business Roundtable, which is a statewide education coalition. Does your state have a Science and Mathematics Coalition? Contact the state director and get a list of the members. Get a list of your department's alumni. Send out a letter to every one of those people. Include a current brochure and your business card. Be sure the brochure looks professional but not too bland. Use light brown or dark gray to distinguish it from all that white paper executives receive. You do not want it to get lost on someone's desk.

And follow up! Make a telephone call even to busy people. You may not get past the secretary, but make it clear that you have something of value to offer. Be careful to write a letter that makes this clear. I was not so careful one year and somehow the people I wrote to thought I was looking for a donation. They wrote back and said they had done their charity for that year! I called them immediately to explain. Your letter and the brochure should mention successful projects and satisfied sponsors (with their permission).

I recommend that you make a lot of visits to potential sponsors. Approach them with the confidence that you have something of value to sell. Get on the telephone and call those to whom you sent letters. Explain briefly that this is a follow-up phone call and ask if you can come and visit. Be prepared to make some suggestions of the types of research questions that frequently arise in industries of their particular type. Remember that many people like to work with students. And many businesses and industries cannot afford to hire all the people they need to investigate all the problems that are on their desks; this service you are offering can satisfy them on both counts. You need to get their attention.

Of course, it helps to have a good track record. That can come with time. I do not mind if, at the start, you use a list of our successful projects to show your potential clients examples of what students can tackle. [See the concluding section of this chapter.] Sometimes the mere suggestion of possible projects will inspire people to get back to me saying, “You know, I think I have a problem you might be interested in. It looks like this....” By and large, if it does not sound off-the-wall, I’ll say, “I’m coming.” And when I come, I try to bring a couple of faculty members with me, people who know enough about the subject to speak intelligently about it. So, for example, I can talk a bit about queueing theory, but we have a person in the department with a Ph.D. in operations research who is far more knowledgeable about that. It is very important for the potential client to feel that the team will have expert leadership. They may be altruistic, but they do want a product for their fee. Don’t be discouraged because you think you know nothing. Many times I have found, after listening to the problem and determining the real underlying question, that I know a whole lot more about it than at first glance. And remember that we are expecting students with an undergraduate mathematics background to solve the problem, albeit with strong mathematical leadership.

Defining the Problem

Let’s say that you get some sign of interest in sponsoring a project. You have to work hard at this point to understand the client’s problem. My experience is that the stated problem is usually too vague and almost so nonmathematical that it does not make much sense as an undertaking for a whole academic year. Recognizing which faculty members might have the expertise to discuss the problem intelligently with the potential sponsor is an important step in defining the problem. My technique worked pretty well. I wrote a paragraph explaining what the problem seemed to be and what the AML might be able to do toward its solution. I gave it to the potential sponsor and a few faculty members. As the sponsor refined his/her thoughts as to what the object of the research really was about, the faculty members and I became more familiar with the terminology of the industry or business. I continued to refine and rewrite a statement of work until we had a proposal that everyone understood and that had a substantial mathematical component that might appeal to mathematics students and faculty members.

It is sometimes necessary to ask if you can start with a small project before going on to a larger one. Here is an example. We were asked to work out a production schedule for seven production lines. We asked if we might start with four. We worked on the busiest four and the client was pleased.

Beware of projects where you have not personally seen the form and content of the data files. We were taken in by assurances on one project for Blue Cross Blue Shield and after we started the project, we realized that their data were not complete. The students had to spend a whole semester collecting information. Though it is a valuable experience for students, it is not the mathematical experience we had hoped for. While some programs seem to be able to handle one-semester projects, I urge against it. There is too much start-up time needed to learn about the nature of the problem. And if something goes wrong, there is little time to correct it. Our project with one company analyzing acoustical data was not as much of a success as I would have liked. The data were digitized acoustical tapes of three types of vehicles. But there was a lot of noise in the data: if we had had the luxury of another semester, we would have asked for more [and better] data. I feel confident that we would have had much better results. There is another lesson in this particular project: the company wanted the results in a hurry because it was making a bid on a job that depended on those results. There was an urgency to their demand that I should have realized at the start and, under the circumstances, I should probably have declined the project.

Choosing Faculty Leadership

Does every faculty member in my department participate? No. Of the 28 tenure/tenure-track faculty members in the department, eight are in mathematics education. Although we have had contact with public school systems who might like our help on a project, by and large, these faculty members have been so involved with in-service institutes and training programs along with other regular teaching duties, that it has been impossible for them to take on yet another responsibility. Regrettably the mathematics secondary education majors, who might benefit from the AML experience, have such tight programs with student teaching and other course work that they are generally unable to participate. Furthermore, there are some faculty members who are against the whole idea of the AML. They believe that students should be taking more formal mathematics courses and they want to be left out of it. I do not want them to be dealing with our students or our clients, so it is best to exclude them from consideration as project leaders. But there are only a very few of those people. The rest of the department is willing to take a chance at it, given that they get a partner who is experienced.

The ideal situation is to have an experienced person act as the project director. Then take someone who is wavering and appoint him/her as co-director. Lots of people will try it, given the summer stipend, the chance to work with really good students, the small class size, and the reduction in the regular teaching load. I will also describe later some projects in which the co-director was from another department. That is another option which frequently adds the expertise you need to complement the director's own background. As with all interdepartmental teaching arrangements, this requires skillful negotiation.

Actually going ahead and deciding on the project(s) you are going to accept can depend on other departmental scheduling and must be coordinated carefully with the department chairperson: who is teaching what, who can be spared from regular courses, the range and quality of students who might be available and interested, and who is in the middle of a large funded project at the time. There is a lot of give and take here. Your department chair must be supportive. We generally have only one project per year. I think that at most schools it will be hard to support more unless you use graduate students as team members or co-directors.

Signing on the Dotted Line

Let's say you have decided you would like to work on a project with Ms. XYZ at Company ABC. Professors Smith and Doe are ready to co-direct the project and everything seems to be falling into place. The next step is to make a brief research proposal to the client. We do this in an informal way: I write the proposal with the help of the potential project directors. We send it to the potential client. Ms. XYZ might make some corrections and send it back. We fine-tune it, being careful to promise to make a study of the problem without agreeing to solve it. We explicitly remind the client what data we expect her to supply and in what form. We state that we will supply an interim report by Christmas and a final report by the end of May. We require that the liaison meet with us during each semester and that they arrange for an oral presentation, usually at the company, at the end of each semester.

We are required to have a formal contract before we begin work. But at this stage, we want to be sure that we have understood the problem and that Ms. XYZ understands what we are offering to do for her company. We had used the Claremont contract and everything was fine, but somewhere along the way, the university decided we should use a state-sanctioned, boilerplated research contract that really does not apply to undergraduate research projects. The state retains rights to patents, publication rights, and many other things. I tell the client to take it to her lawyer and eventually her lawyer talks to our lawyer and so on, back and forth, until everyone is satisfied. That's what they get paid for! Meanwhile I begin trying to find a suitable student team.

Confusing legal issues do arise, so I urge you to allow your lawyers to look over things. For example, one AML team wrote a computer adaptive placement test for the university [the client] based on a selection of problems from the MAA. The problems were available to us from the MAA when we joined the placement test program and we could use these questions in any way we liked. But what if the MAA decided that it wanted to market our computer adaptive test? What would belong to the MAA and what would belong to Towson University? The intellectual property policy of the university and the policies of the MAA may be in conflict. It is best to have clear contracts wherever possible.

Confidentiality

At every stage, one should be aware of the confidential nature of conversations with potential and current clients. In approaching the contract stage, we include an understanding about the proprietary nature of the work of the sponsor. We state clearly that if there are conflicts of interest [spouse works for competitor, student is employed by competitor, *etc*.], we will inform the client. Furthermore, company data belong to the company and we are not at liberty to reveal them to anyone. They must be protected when they are in our possession. It is imperative that we have a room in which the team can work in an unimpeded way and in which it can leave records and calculations in a secure environment. We discuss this aspect of the consulting enterprise in detail with our students and faculty. We take this very seriously and we communicate our commitment to security to the entire team and the department.

Since we work in the Baltimore-Washington, DC, area, we frequently come across potential AML work that requires U.S. citizenship. While almost all of our faculty are U.S. citizens, that is not always true of our best students. I urge vigilance on this. I have been embarrassed on one site visit when one of my colleagues was asked to wait outside the gates because he did not have citizenship papers. We all left at that point. Ask ahead if you think this might be a problem.

Fees

Money is supposed to change hands on these projects. We do not plan to make money, only to cover our expenses and have some development funds for new equipment and overhead. We come cheap compared to the Math Clinic at Harvey Mudd College. We have tried to pare down the budget as we have found that corporate generosity is more limited now than ten or even five years ago. I will mention some ways to further reduce expenses, but each such step changes the nature of the enterprise. Here is an approximate budget:

Faculty summer stipends @ $4000 each	$ 8000
Assigned time: 3 hours/semester/faculty member	8400
Assigned time for AML director: 3 hours/year	2100
FICA, etc., on salaries [approx. 8% of subtotal]	1500
Printing, computing, overhead	1500
GRAND TOTAL	$21500

We do not buy software for clients. If we need the software, the expense is borne by the company and the sponsor retains the software after the project. Although we almost always write software for a client nowadays, we do depend on big commercial packages in mathematics, operations research, and statistics for many applications. The university has site licenses for most of what we need. Since we can be linked electronically, the company can share necessary software it already owns.

We collected approximately this amount for many years, but of late, we have found it much harder to demand this price. The primary barrier is that many home offices of companies once in the Baltimore area have moved to other cities because of corporate mergers and buy-outs. This leaves a lot less cash under the control of local officers for research and development purposes.

How to cut expenses? Nothing is easy, as we were already at bare bones. But we have provided slightly less summer money for faculty and we have reduced our overhead by limiting the amount we put aside for future office and computer supplies. Eventually this may have deleterious effects. For now, we seem to be coping by drawing on departmental operating funds. We now have a Master's program in Applied and Industrial Mathematics that requires an internship. While many students fulfill the requirement by working at a business, industry, or government installation, we encourage some qualified graduate students to serve an internship as the co-director of an AML project. Their graduate assistantships help to compensate them for their time. Other expenses, such as student travel and presentations, can come from a university student research fund.

Choosing Students

We consider the invitation to work on an AML project to be an honor. We ask faculty in the Mathematics Department to recommend potential student-team members. Students generally are entering their junior or senior year and should have taken a large number of courses, though the nature of the project determines which particular courses are necessary prerequisites and which are preferred. It is also common that we need a specific skill that our own majors do not have. On one project for the Maryland State Income Tax Division, we needed students who had taken advanced auditing classes and also some statistics courses. We went to the Accounting Department for their recommendations. Computer science students are frequently invited to join a project team. On an analysis of sound signatures of three types of vehicles, we needed the help of a physics professor who is an expert in acoustics and tapped some of his students to join the team.

We invite students to join a project team at about the time they are planning their schedules for the next semester. We do not encourage students to substitute the AML for a course that they should be taking as part of their major. We think that the AML is an honor and should be taken in addition to the standard major requirements. There are times when we have relented though. If the course the student needs is in operations research and the project is heavily involved with linear and nonlinear programming, sensitivity analysis, and model verification, we may allow the substitution.

While students enroll in the semester course for three credits, we carefully explain the commitment as quite a bit more extensive and intensive. Many of our students work outside of school. We make it clear that they cannot carry a full load of classes including the AML and work as much as 20 hours a week. Occasionally a student will have to decline our offer for financial reasons, but this happens rarely.

What students make the best team members? Of course, we want bright students, but not necessarily those with the highest GPA's. We need a variety of talent, so we look for those skills and backgrounds that seem best suited to the project at hand. We must be sure that they can work as part of a team, meet deadlines, work independently at times, communicate well in both written and oral media. They need a strong work ethic to get through the pressures of their other courses when the project is at its most demanding. We want people who have a serious respect for responsibility and integrity. And we have not been disappointed in the teams we have chosen over the years. Who are the worst student team members? The A students in mathematics and computer science who are loners, who work best alone, resent having to explain themselves to other students, or who write programs that no one else can understand and then cannot be bothered with documentation. Needless to say, there is no room for sexist or racist behavior; students who cannot treat others of different backgrounds with respect are unwelcome.

Team sizes vary, but I find the best teams have six members. Why? The group will be divided into pairs or into threes to accomplish subtasks. Suppose all the students know a little discrete mathematics, but we are looking at the utility of graph theory at some stage in the modeling process. Perhaps two students know a little more graph theory than the others; they are sent off to do some reading and to teach the next class. Meanwhile another pair may be doing some statistical analysis on data and another pair is starting to write up an account of the beginning assumptions and methods used. They formally come together in class more often in the beginning of the semester (usually three hours a week) and, as the semester progresses, they may meet less as one group and a lot more in their subgroups. We expect that the students will work in small groups and with the faculty to familiarize themselves with relevant material and techniques and then teach their teammates. As a group they have to decide if their proposed method of solution is proving fruitful and seems accurate.

Table 1. Timetable of projects

Timetable of a Project	
Spring year $n-1$	• solicit project and define work for year *n* • select faculty members to direct project for year *n* • solicit and select student team for year *n* • sign contracts for year *n*
Summer year $n-1$	• faculty members work with company liaison to further define project for year *n* and secure data
Fall year *n*	• students and faculty begin work, meet at least 3 hours/week as a group before breaking into smaller subgroups further into semester
Midsemester year *n*	• team presents **interim** oral report to sponsor • new students may be selected for second semester
December year *n*	• team presents **formal written and oral report** to sponsor, usually at sponsor's workplace • sponsor pays for first semester deliverable
January year *n*	• new students on team get up to speed by working with faculty and other team members and reading report
Spring year *n*	• team continues work, perhaps with new direction resulting from first semester report and feedback from sponsor • solicit new project and define work for year $n+1$ • select faculty members to direct project for year $n+1$ • solicit and select student team for year $n+1$ • sign contracts for year $n+1$
May year *n*	• team finishes project for year *n* • team presents **final written and oral report** to sponsor at sponsor's workplace • sponsor pays for second semester deliverable

As the Director of the AML, I insist that the team make several oral presentations during each semester [See Table 1]. I attend, and we encourage other faculty members on the AML Committee to attend so that the interim reports and the final reports at the end of each semester are well rehearsed and so that we can discuss the mathematical modeling approach being used by the team. We do offer advice to the team on the mathematical strategies they might try, and oftentimes we find that our naiveté is helpful! We ask simple questions about the nature of the problem and the scope of the assumptions that help to further clarify the model and the technique. This is an extremely important part of the learning process for the team and for other faculty members. Our teams are always shaky when they first do their oral presentations, but by the end of the semester, the students are poised and professional and frequently the sponsor is amazed at their ability to give a quality professional report.

At the mid-fall semester presentation, the liaison may very well express concern at the direction in which the research is going. This may have several causes. For example, the liaison may need more information about the mathematics. But frequently it is the team that has missed some essential fact about the industrial problem. Such information may have been omitted by the sponsor just because it did not seem pertinent. When the model is presented by the team, things may not look right to the practitioner. The model does not seem to model his/her experience. This correction stage is one of the most essential steps in the process. We are inexperienced with the kind of practical and everyday knowledge of the industrial workplace. If we have omitted a variable or parameter that we need, it is important to find out early in the modeling process. Even if the students are weak with their presentation skills at this stage, do not skip this step with the liaison. It may save your project from failure.

Monitoring Progress and Changing Course

While the faculty have a pretty good idea of how the problem should be solved by the time the students come on board in September, the AML Director should take an active role in monitoring progress and attending the class occasionally. There is not a lot of time to do the work. An academic year sounds like a long time, but school begins in September, we have a break for Thanksgiving and suddenly it is Christmas and final exams. A written and printed final report as well as an oral report should be presented to the sponsor before the team leaves at the end of the first semester.

As mentioned above, the middle of the year can be a time for reflection and reevaluation of assumptions, goals, and methods. The Director should be perceptive and open about the need for recharting the course and should provide support for the team and the faculty as they make necessary changes. When some demands cannot be met, the Director may have to insist that the sponsor accept restrictions on the scope of the project.

Sometimes we need new students on the team in the second semester. Graduating seniors leave, students may leave to take other courses, or we may choose to add students with different skills because the skills we need in the second phase may be different from what we needed in the first phase. Flexibility is a key to management of these projects.

We have a January minimester and students in the AML are not required to come in to work then, but many team members choose to do that anyway. Those coming to the project for the first time are asked if they would be willing to come in during January to catch up. Nevertheless the official beginning of the semester is about February 1, spring break occurs in March, and before you know it, it is time for finals and commencement. Under no circumstance do you want to leave any substantive work for students for the beginning or end of May. Graduating seniors tend to represent the majority of the AML team in the spring, so their minds and interests may begin to wander after the end of April. The final report and the oral presentation should be polished and represent the team and the institution in the best possible light. This takes a lot of preparation, correction, and polishing. Do not leave it until the last minute.

To our great horror one year, we finished the project and proudly presented the results to the company only to be told that the model was wrong. It did not conform to the long-term experience of the management. What could have gone wrong? The faculty and I sat down with the sponsor for several hours and discovered that though they had read the assumptions about the manufacturing process that we had asked them to approve, they had never mentioned some important fact about treatment of inventory that they wrongly assumed we understood. They had never mentioned this before. Students were dispersing for the summer, faculty were ready to go on to other projects. And I was distraught. The solution was amicable. The sponsor paid for another semester and we went at it again with the new assumption built in. The *final* final report went very well and the project provided the sponsor with a correct and viable solution.

The Writing Process

Writing a report of the type we present to sponsors would have been extremely difficult in the days before word processing. At least one member of the team must be charged with starting to write the report on the first day of the first semester. The report should contain an executive summary (done last), an introduction to the problem, the statement of the research project, an introduction to the mathematical techniques that are to be employed, a discussion of the results of the modeling process, an analysis of the strengths of the model, the results and the implications for the sponsor, and further research questions. Software and proper documentation should be supplied as well.

For many years, I asked someone from the English Department familiar with technical writing standards and techniques to come to speak to the team. Now most of our students have taken the course in Business and Technical Writing, and we look at their grade in the course before accepting them into the AML. Nevertheless, the faculty should closely monitor the writing style and the accuracy and level of the mathematical content in the report. The Director of the AML should take an active part in reading and proofreading the manuscript. At all times, the audience, company executives, should be considered in choosing the mathematical content and level. Engineers may be able to handle more highly technical material than marketing executives, so the team must adjust accordingly. The sponsor wants to understand what the team did to obtain its results, but if the report is too technical and difficult for the lay person to understand, it accomplishes little in this regard.

What Works for Us

The most popular tracks in our department are in actuarial science, applied mathematics, and the double major in mathematics and computer science. This gives us a choice of some great students who have a business background with their mathematics, including courses in management, accounting, and finance. Others are terrific programmers and software or database experts. These students make excellent team members. They can go on to get great jobs.

One of our projects dealt with computer security, and the two project directors were faculty in mathematics and in computer science. This has led to many more collaborations among the faculty, leading to NSF funding for their joint work, a new track in computer security within the computer science major, and a similar option for students in the double major in math and computer science.

The best projects for us are those in business and industry as well as in government with less engineering and more environmental science, business, and management. We find that all of our projects rely heavily on statistical analysis and many involve operations research. In the list of projects in the following section, I have given a brief abstract of each problem.

But each institution needs to find its niche.

The economy in a given region can make a huge difference in how a program like this can be run. We have had to become adaptable to the situation in the Baltimore area. Right now, we are involved with county and city agencies that have great projects but no means to pay. While we do a wonderful public service, we need some financial support or we will soon be *kaput*.

AML did a project for the Baltimore City Fire Department last year. The team was to come up with a solution to their overtime problem: Too many personnel coming in on overtime pay, costing many millions. How could we redefine their scheduling process with the many constraints imposed by the level of work required, union rules, and so forth? The team was chauffeured by the Fire Department van whenever it went downtown to meet with officials. That added a bit of excitement on campus! AML gave the final report in the CityStat Room to the mayor and his cabinet. Fox News did a piece on the presentation and gave press to the results on saving money on overtime. The students got a tour of City Hall and the mayor's office and they have done several presentations in undergraduate research venues since then. No money changed hands, but we did get publicity!

Energy and enthusiasm on the part of the Director will go a long way to making the program viable. You need the *chutzpah* to walk into executive offices and sell the product. It is a lot of fun and students have uniformly agreed that it was a highlight of their undergraduate experience.

Applied Math Lab Projects at Towson University

2003–04 Baltimore County Department of Environmental Protection and Resource Management
The Applied Mathematics Laboratory worked with the Baltimore County Department of Environmental Protection and Resource Management to analyze water well failure in rural Baltimore county.

2002–03 Baltimore City Fire Department *An Analysis of the Optimal Staffing Level of the Baltimore City Fire Department*
This project determined an approximately optimal staffing level for the Baltimore City Fire Department.

1998–99 Science Applications International Corporation (SAIC) *Validation and Enhancement of Applications of Models from Epidemiology to INFOSEC Assurance Metrics*
This project investigated the use of epidemiological models to simulate and analyze the spread of computer viruses through a corporate network.

1996–97 Bell Atlantic Network Engineering and Capacity Management *Customer Usage Profile for Fast-Packet Frame Relay*
The AML team developed customer usage profiles for the relay of fast packets through the communications network. Data communications engineering and queueing theory were used to develop models for projection of network impact, processor occupancy, and link utilization in the network serving Bell Atlantic's industrial and business clients.

1995–96 Towson University, Academic Services *A Computer-Adaptive Mathematics Placement Test*
The AML adapted a computer-based mathematics placement test from the nationally recognized series of tests available from the Mathematical Association of America. The AML test is computer-adaptive and uses information about the student's background to customize the level of the test and adjusts as the student takes the examination. Students are advised on placement in accordance with their skill level and their major.

1993–94 Martin Marietta Aero and Naval Systems *Interface Between Database System M/Vision and Statistical Software Stat17*
The AML team wrote a front and back end to a statistical package linked at the laboratory bench to a materials database used in the production of state-of-the-art marine and air craft.

1990–92 Becton Dickinson Microbiology Systems *Scheduling Production of Prepared Plated Media*
Plates treated with growth media used for cultivation of bacteria used in disease diagnosis are produced on several production lines at the Becton Dickinson Hunt Valley installation. The scheduling package designed by the AML optimizes the weekly schedule and takes account of inventory, quarantines, cleanup and setup times on three of the busiest lines.

1989–90 Towson University, Academic Services *An Enrollment Model for Resource Scheduling*
An AML team developed software to help the Registrar's Office schedule courses based on the changing demands within majors and in requirements for degrees.

1988–89 Blue Cross Blue Shield of Maryland *Statistical Survey of Long-Term Care Patients in Nursing Homes*
Towson actuarial science students and other mathematics majors investigated the length-of-stay of Marylanders in nursing homes for the purposes of helping the sponsor evaluate the marketing and pricing strategy for long-term care insurance policies.

1986–87 State of Maryland, Comptroller's Office, Retail Sales Tax Division *Estimation of Sales Tax Liability*
Calculation of the liability of sales tax to be paid to the State of Maryland by retail companies doing business in the state had been obtained by teams of auditors combing through all of the business' records. The AML team wrote software to allow for just one or two auditors to select a stratified sample to estimate the tax liability with a high degree of accuracy.

1985–86 Citicorp of Maryland *Customer Service Queuing Model*

The Citicorp Choice credit card service desk had varying needs for operators and telephone lines. By setting up a real-time scheduling device, the AML team helped Citicorp meet its service goals on the help-line.

1984 A.A.I. Corporation *Acoustical Pattern Recognition*

Students in physics and mathematics joined together to study the acoustical signatures of several vehicles in an attempt to design a remote recognition device.

1983–84 Union Trust Company of Maryland, Operations Center *The Collection of Cancelled Checks for Processing*

Union Trust had 85 branch offices in all parts of Maryland. The AML team developed an algorithm for the collection of cancelled checks from the branch offices for processing at the central Guilford Avenue center. The system was written to take into account the volume of check activity at branch offices, the monetary value of the checks, and the efficiency of routes.

1980–81 Towson University, Auxiliary Services *Student Housing at Towson University*

The student-faculty team investigated demographic and enrollment trends and projected the on-campus residential needs for Towson. The projections were used in presenting the funding package to the legislature.

Martha Siegel works at a large state-supported university which serves both residential and commuter students. Towson offers undergraduate programs in both the traditional arts and sciences and applied professional fields, as well as several graduate-level programs.

Project Deliverables I

Robert Borrelli
Harvey Mudd College

I can give a little perspective on the list of reports which follows. One of these is reproduced in its entirety in the Appendix. Reports are pretty thin because we don't believe in putting in lots of stuff. Most of the work has already been done and given to the client. All of the software has been installed and the documentation written. The final report is more of a formality which brings everything together to make it look nice, something that the liaison can take to the boss and say, "Here's the conclusion of the project." We bind it and send it out to the client who puts it on the shelf.

Let me offer a quick overview of the chronology of the projects that we've had in the past followed by a few war stories. The projects are listed chronologically. These are the clients we've had over the years and these are the projects that we've had from those clients.

Harvey Mudd Mathematics Clinic Projects By Date

1973–1974 Bell and Howell: *Analysis of Scintillation Phenomena in Rear Screen Projectors* (HMC); **Northrop**: *A Kinematic Handbook for Missile Design* (HMC).[1]

1974–1975 Chevron: *Models for Abnormally High Fluid Pressures* (HMC); **LA County Superior Court**: *Juror Utilization Problem* (HMC); **Rockwell**: *Mathematical Modeling of Air Pollution Transport* (CGU).[2]

1975–1976 Becton, Dickenson, & Company: *A Study of Quantitative Methods in Market Forecasting* (HMC); **Ameron, Inc.**: *Computerized Coatings Formulator* (CGU); **Atomics International**: *Fuel Element Life Performance Model* (CGU); **Chevron**: *A Mathematical Model of Compacting Sediment with Diagenesis* (HMC); **Honeywell Marine Systems, Inc.**: *Development of Trainer Display Programs* (HMC).

1976–1977 Atomics International: *Fuel Element Life Performance Model* (CGU); **Jet Propulsion Laboratory**: *Design of a General Purpose Photo Composition System* (HMC); **Pomona Valley Municipal Water District**: *Predicting Groundwater Nitrate Concentrations* (HMC); **California State Architect's Office**: *Life Cycle Costing* (HMC); **Chevron**: *Abnormally High Pressure in Sedimentary Basins* (HMC); **Honeywell Marine Systems, Inc.**: *Digital Feedback Control of Targeting Devices* (CGU).

1977–1978 Jet Propulsion Laboratory: *Search for Extraterrestrial Intelligence: Data Processing Techniques* (CGU/HMC); **Chevron**: *Supernormal Pressures: ETC* (HMC); **General Dynamics**: *Analytic Model of the Concentration of CO2* (CGU); **Pomona Valley Municipal Water District**: *A Method for Predicting Groundwater Nitrate Concentrations Vol. II* (HMC); **U.S. Forestry Service**: *Mathematical Simulation of Chaparral Management Alternatives* (CGU); **TRANSANA**: *Multivariate Data Analysis*

[1](HMC) Harvey Mudd College
[2](CGU) Claremont Graduate University

(CMC); **Office of Naval Research**: *Simulation, Confidence Bounds, and Goodness-of-Fit for Mixed Exponential Distributions* (CMC).[3]

1978–1979 General Dynamics: *Team Games: Cooperative Effects* (CGU); **General Dynamics**: *Parameter Optimization for a Homing Guidance System* (CGU); **Aerospace Corporation**: *Efficient Numerical Methods for Solving Differential Equations of Rocket Flight* (HMC); **Interstate Electronics**: *Automatic Word Recognition I* (CGU); **U.S. Forestry Service**: *Estimating the Cost of Fires in Chaparral Lands* (HMC); **U.S. Forestry Service**: *Computer Algorithms for the Comparison of Chaparral Land Management Alternatives* (CGU); **Office of Naval Research**: *Confidence Bounds for Reliability* (CMC); **Harvey Mudd College**: *Cash Flow Management in a Small, Private College Environment* (HMC).

1979–1980 U.S. Forestry Service: *Probability Distributions for Fire Occurrence* (HMC); **U.S. Forestry Service**: *Strategies for Chaparral Management* (CGU); **Interstate Electronics**: *Automatic Word Recognition II* (CGU); **Harvey Mudd College**: *Cause Computer Net* (HMC); **Aerospace Corporation**: *Multivariable Interpolation for Aerodynamic Problems* (HMC); **Atomics International**: *Evaluation of Nuclear Safety Code Efficiency Schemes* (CGU); **General Dynamics**: *Compiler Development* (HMC); **General Dynamics**: *Parameter Optimization for a Homing-Guidance System* (CGU); **Rand Corporation**: *The Bomber Prelaunch Survival Problem* (HMC); **Rockwell**: *Evaluation of Nuclear Safety Efficiency Schemes* (CGU).

1980–1981 Rockwell: *Nuclear Safety Code Study* (POM/CGU)[4]; **Rand Corporation**: *Stability Analysis of a Passive Communication Satellite* (HMC); **General Dynamics**: *Homing Guidance Parameter Optimization with Nonlinearities* (CGU); **Aerospace Corporation**: *Construction of a Data Base for Satellite Design* (HMC); **Claremont Graduate University**: *Facilities Management Model* (CGU/CMC); **Interstate Electronics**: *Automatic Word Recognition III* (CGU); **Lockheed**: *Computational Aerodynamics I* (CGU); **Megatek Corporation**: *Color Presentation Transparencies* (HMC); **General Dynamics**: *Modeling, Forecasting and Planning for Research and Development* (HMC).

1981–1982 Lockheed: *Computational Aerodynamics II* (CGU); **Interstate Electronics**: *Automatic Word Recognition IV* (CGU); **General Dynamics**: *Forecasting and Planning for Research and Development* (HMC); **ARCO Gas & Oil**: *Evaluation of Numerical Dispersion and Grid Orientation Effects for the Two-Phase Immiscible Displacement Problem* (HMC); **Texas Instruments, Inc.**: *Synthetic Speech Listener* (HMC); **U.S. Forestry Service**: *Forest Fire Suppression* (CMC/CGU); **U.S. Forestry Service**: *Network Analysis of Fire Management* (POM/CGU).

1982–1983 Texas Instruments, Inc.: *Automatic Pitch Determination for Human Speech* (HMC); **ITT Barton**: *Analysis of a Vibrating Element Densitometer I* (CGU); **Jet Propulsion Laboratory**: *Applications of Correlation Techniques for Battlefield Identification* (CMC/CGU); **Teledyne Microelectronics**: *Computerized Transient Thermal Analysis Hybrid Circuits I* (CMC).

1983–1984 U.S. Forestry Service: *A Heat Transfer Model for Prescribed Fires in Southern California Brush* (POM/CGU). **Teledyne Microelectronics**: *Computerized Transient Thermal Analysis of Hybrid Circuits II* (HMC); **Jet Propulsion Laboratory**: *Applications of Correlation Techniques for Battlefield Identification I* (CMC/CGU); **ITT Barton**: *Analysis of a Vibrating Element Densitometer II* (CGU); **General Dynamics**: *Electromagnetic Propulsion* (HMC); **NASA Research Center**: *Depiction of Simultaneous Maneuvering of Two Moving Aircraft* (HMC); **NASA Research Center**: *Three Dimensional Computer Graphic Display Of The Dryden Valley* (HMC).

1984–1985 General Dynamics: *Radar Design Using Symbolic Manipulation Software* (HMC); **Honeywell Marine Systems, Inc.**: *Mathematical Foundations of Realistic Video Simulations* (HMC); **Jet Propulsion Laboratory**: *Parameter Extraction and Transistor Models* (CGU); **Jet Propulsion Laboratory**: *Application of Correlation Techniques for Pattern Recognition* (CMC/CGU); **Pacific Bell**:

[3](CMC) Claremont McKenna College

[4](POM) Pomona College

Telephone Database Information System (HMC); **Perkin-Elmer Corporation**: *Identification Analysis in Mass Spectrometry* (CGU); **U.S. Forestry Service**: *Modeling Heat and Moisture Transfer for Prescribed Fires in Southern California* (POM/CGU).

1985–1986 Perkin-Elmer Corporation: *Computational Methods for the Mass Spectrometer* (POM/CGU); **Jet Propulsion Laboratory**: *Modeling Short Channel MOSFETs for use in VSLI* (CGU); **Jet Propulsion Laboratory**: *A Probabilistic Position-Fixing Model* (CMC/CGU); **Honeywell Marine Systems, Inc.**: *Generating Realistic Video Displays by Manipulating Photographic Images* (HMC); **Garrett Automotive Products Co.**: *Mathematical Modeling of Floating Ring Bearings in High Speed* (HMC); **System Development Corporation**: *Multispectral Classifier* (HMC).

1986–1987 U.S. Forestry Service: *Influence of Multiple Fires on Suppression Effectiveness in California* (CGU); **General Dynamics**: *Development of a Code Analyzer for Real Time Software* (HMC); **General Dynamics**: *Mathematical Modeling and Simulation of Neural Network Image Classifier* (HMC); **Jet Propulsion Laboratory**: *Prediction of Contact Yields for MOSFETs* (CGU); **Jet Propulsion Laboratory**: *Identification and Targeting of Military Units in the Battlefield* (CMC/CGU); **Rand Corporation**: *Optimal Strategies for Regional Defense* (HMC).

1987–1988 Jet Propulsion Laboratory: *Benchmarking Time Warp Synchronization Mechanism* (HMC); **Jet Propulsion Laboratory**: *Optimal Data Collection for MOSFET Modeling* (CGU); **General Dynamics**: *Modeling and Simulation of Neural Network Image Classifiers* (HMC); **General Dynamics**: *Multipath Modeling* (CGU); **Aerojet Electrosystems**: *Outgassing and Contamination in Vacuum Systems* (HMC); **NASA Research Center**: *Flutter Stability of Aircraft in Flight* (HMC).

1988–1989 NASA Research Center: *In-Flight Aeroelastic Stability Parameter Estimation* (HMC); **Chevron**: *Parameter Studies for a Model of Abnormal Fluid Pressure* (HMC); **General Dynamics**: *Modeling and Simulation of Neural Network Image Classifiers* (HMC); **Jet Propulsion Laboratory**: *Applications of Correlation Techniques* (CMC); **Northrop**: *An Investigation of Reflection, Transmission and Absorption of Electromagnetic Energy* (HMC); **Teledyne Microelectronics**: *Establishment of a Basic Statistical Process Control Program* (HMC); **U.S. Forestry Service**: *Statistical Analysis of Indices for the Evaluation of Forest Fire Potential* (CGU); **Jet Propulsion Laboratory**: *Linear and Non-Linear Estimation* (CGU); **Jet Propulsion Laboratory**: *Robust Experimental Design* (CGU).

1989–1990 Information Sciences Institute: *Heat Transfer in Transistors* (CGU); **McDonnell Douglas**: *Density and Energy Distribution of Electrons in the Lower Van Allen Belt* (HMC); **Northrop**: *An Investigation of Reflection, Transmission, and Absorption of Electromagnetic Energy Interacting With Waveguides* (HMC); **Rand Corporation**: *Theory of Chaos Applied to Combat Models* (HMC); **Teledyne Microelectronics**: *Establishment of a Statistical Process Control Program* (HMC).

1990–1991 Rand Corporation: *Chaos In Combat Models* (HMC); **Jet Propulsion Laboratory**: *Software Metric Data Analysis* (CGU); **Information Sciences Institute**: *Heat Transfer From a VLSI Chip* (CGU); **McDonnell Douglas**: *Low Altitude Proton Currents Due To Solar Cycles* (HMC); **Lockheed**: *Fault Tree Analysis* (CMC); **Hughes Aircraft**: Automated Database Generation (HMC); **Hughes Aircraft**: *Resource Allocation Model* (CMC); **Teledyne Microelectronics**: *Statistical Process Control In Microelectronics* (HMC).

1991–1992 Harvey Mudd College: *Instructional Module On Chaos* (HMC); **Information Sciences Institute**: *Modeling Of Short Channel MOSFET Devices for Use in VLSI Simulations* (CGU); **Office of Naval Research**: *Sizing An Artificial Neural Network* (CGU); **Chevron**: *3D Seismic Image Data Compression* (HMC); **Jet Propulsion Laboratory**: *Knowledge-Driven Hypermedia: An Encyclopedia of Software Components* (HMC); **Northrop**: *Maximizing Energy Attenuation in Coated WaveGuides* (HMC).

1993–1994 AlliedSignal Automotive: *Radial Wheel Thickness Optimization* (HMC); **Argonaut Insurance Company**: *Graphical Representation of DNA Hybridization* (HMC); **HSC Software**: *A Self-Contained Design Environment for Creating Images with Ultra-High Realism* (HMC).

1994–1995 **Beckman Instruments**: *Simulation of DNA Array Hybridization* (HMC); **AlliedSignal Automotive**: *Optimization of Blade Thickness Distribution for a Turbocharger Wheel* (HMC).

1995–1996 **Golder Federal Services**: *A Transport Pathways Algorithm for the Repository Integration Program* (HMC); **Grumman B-2 Division College**: (HMC); **Environmental Systems Research Institute, Inc.**: *From Here to There: Finding a Short Path Quickly with Limited Memory* (HMC); **Eaton Corporation**: *Strategic Forecast for the Powertrain Market* (HMC); **3Com Corporation**: *Dynamic Optimal Inventory Modeling of Finished Goods* (HMC).

1996–1997 **Bank of America**: *Java Based Library for Interactive Banking* (HMC); **Beckman Instruments**: *Classification of Serum Protein Electrophoresis* (HMC); **Fair, Isaac & Company, Inc.**: *Understanding Neural Net Models* (HMC); **Jet Propulsion Laboratory**: *Formal Methods Applied to Spacecraft Subsystems* (HMC).

1997–1998 **Los Angeles Police Department**: *Police Record Searching Project* (CMC); **Fair, Isaac & Company, Inc.**: *Recognizing Patterns in Data* (HMC); **Bank of America**: *GUI Development of Context-Sensitive Modular Components* (HMC); **Aerojet Electronic Systems Division**: *Multiple Hypothesis Tracking (MHT)* (HMC); **Beckman Coulter**: *Algorithm Collection Project* (CGU); **Environmental Systems Research Institute, Inc.**: *Cluster Identification and Data* (HMC); **Hughes/Raytheon**: *Availability Analysis of a GPS Satellite Navigation System* (CGU); **Information Sciences Institute**: *MOSFET Device Modeling* (CGU); **Los Alamos National Laboratory**: *Adaptive Methods for Accelerating Monte Carlo Convergence* (CGU); **Naval Surface Warfare Center, Bethesda, MD**: *Estimating Posterior Defect Probabilities for a System Used in Nondestructive Testing of Composite Materials* (CGU).

1998–1999 **Environmental Systems Research Institute, Inc.**: *Edge Partitioning and the Chinese Postman Problem* (CGU); **Fair, Isaac & Company, Inc.**: *Intelligent Techniques for Scanning and Extracting Information from Text* (HMC); **Owen Racing Shells**: *Optimal Design of a Racing Shell* (HMC); **Science Applications International Corporation (SAIC)**: *Wavelet Transform Detection and Classification* (HMC) (joint eng/math); **Space Systems/LORAL**: *Analysis System for Satellite Propellant Budget Dispersion* (HMC).

1999–2000 **Fair, Isaac & Company, Inc.**: *Preemptive Offers for Portfolio Defense* (HMC); **HNC Software, Inc.**: *Link Analysis* (HMC); **Space Systems/LORAL**: *Advanced Analysis System for Satellite Propellant Budget Dispersions* (HMC).

2000–2001 **Applied Biosystems**: *Modeling of TaqMan Reactions for DNA/RNA Quantification Using PCR (polyermase chain reaction)* (HMC); **ETEC Systems, an Applied Materials Company**: *Statistics of Error Budget for Laser Lithography Systems* (HMC); **Fair, Isaac & Company, Inc.**: *Preference Analysis of E-Commerce Webs* (HMC); **Space Systems/LORAL**: *Optimization of Orbit-Raising Using Ion Propulsion* (HMC).

Summer 2001 **ViaSat, Inc.**: *Elliptic Curve Cryptography* (HMC).

2001–2002 **Fair, Isaac, & Company, Inc.**: *Optimizing Dynamic Online Survey* (HMC); **Northrop Grumman**: *Efficient Satellite Orbit-Raising in 3D Using Ion Propulsion* (HMC); **ViaSat, Inc.**: *Elliptic Curve Cryptography Scheme for Asymmetric Key Generation* (HMC).

We've had a wide variety of clients. Let me say a little bit about the Allied Signal project (1994–95). That was spectacularly successful. You wouldn't have suspected mathematicians would do this kind of work. This is a manufacturing firm which makes turbo chargers for clients all over the world. Originally they were manufacturing turbo chargers for aircraft. These were large, well-designed turbos, but when turbo chargers were installed in cars, they had to down-size them. The instabilities in the design thereby became apparent because the turbos had to spin faster. A lot of really careful modeling went into the design

Figure 1. HMC Math Clinic group

of these turbo chargers and how they were manufactured. For example, they used a computer driven lathe which had to carve out the right shape on the turbo charger's blades.

Interestingly, every one of Allied Signal's clients is a special case: there is no one thing that they can put on the shelf and sell to a lot of clients. This was just ready-made for us. In this case, we created a tool that their engineers could use in the design of their products. So we designed a tool that simulated the behavior of a turbo-charger, given a certain shape of the blade. They already knew basically the generic shape of the blade. The only variables were the thickness of each individual blade and the distribution of the thickness over the blade's surface. Here's the problem: if the blade is too thick at the base, it will be very stable and it will survive the stress of revving it up; however, the resonant frequencies will tear it apart. If it's too thin, you eliminate the resonance problem with the blade, but then the stresses will tear it apart.

So they have several performance characteristics that they need to design for. The question is what should they do in the way of the thickness? So the team worked on that and came up with a very nice software package that would actually do the stress analysis and the resonance analysis, and the whole procedure was automated. What had the company done before? It was incredible. If they had a certain design spec, they would go and look up one of their old turbo-chargers which was almost what the client wanted, and they would modify it. Then they'd build it and bench test it, which meant that they'd stick it on the bench and rev it up to see how long it took to break apart, and they would make certain measurements. If it didn't work, then they'd go back and modify it. They kept repeating this process, going back and forth until they got something that was about right. It took a long time and it was very expensive. So what mathematicians could do, and did do, was to create a tool for them that simplified the whole process. What we actually delivered was a software package with a front end that made it very easy for their engineers to input whatever the custom-designed characteristics should be and a post processor that would display all the results that came out. Then, in between, there were some automatic features that would just do a search to give you the most likely values or the thickness distribution.

This was a hard problem because you had to triangulate the surface, and to keep track of all those points you had to use MATLAB. This brings me back to one of Martha's observations. We thought that the company had MATLAB, and in fact, most divisions in the company did. But when we were all done, after having developed everything in MATLAB, it turned out that this one particular division did not have MATLAB, and they wanted us to do it on a PC. We were done with the project. It was already April. We couldn't redo all this stuff to run on a PC, using a DOS program or something.

What to do? Well, the company refused to buy MATLAB. They said that it was our responsibility.

If we had this project already done in MATLAB, then we would have to provide them with MATLAB. When I checked into it, I discovered that the industrial price of MATLAB was very high, so I asked if I could pay the academic price and then give it to Allied Signal so that, when they renewed the license a year later, the company would pay the industrial price, but MATLAB wouldn't budge. There was one price for MATLAB if it was going to industry. Where was it going to be used? Who was going to hold the license? If Allied Signal was going to hold the license, then it was the industrial price you had to pay. I ended up buying it anyway and giving it to our client. The VP of manufacturing for the company is on our clinic advisory board, he likes us a lot, and he gives us projects. In fact, he prefers us to the engineers, and he's very vocal about his support. Now I love to hear things like that when we're in a big group, and so I bought the industrial version of MATLAB and gave it to them. That's my little war story about software.

Beckman Instruments, another of our clients, is located in Fullerton, right down the street from us. The issue is the same: designing a tool. And again, that's what mathematicians are good at: designing tools that will save the engineers a lot of time. Beckman technicians take samples of DNA which they sequence until they produce enough stuff to put on a slide. Then they create an optical pattern from that slide. What they wanted to do was to analyze the genetic components in the DNA, especially as they related to diseases. At the time, this all seemed far in the future, but we worked on that problem and actually produced a tool that would do a pattern recognition analysis of one of their samples to try to identify what it might be with an associated probability. You could never be certain about any of the results, so you could only do this in a certain confidence interval. We did this for two years, and then we got a project from Beckman which was to classify serum protein. Essentially they would like to be able to identify the proteins available in a sample. We decided to use neural nets for this, rule-based neural nets, which neither I nor anyone else in our shop knew anything about. And that's another point that I wanted to make: we're not all experts from the start. Generally speaking, at the beginning of a project we do not know a great deal about it. But we have confidence in ourselves and in our ability to learn, and that's why I like to have a whole year for a project. We spend the first semester making ourselves experts in this area. So that's what we did our first semester for Beckman. We learned everything we could about neural nets. I had half a dozen papers from Rodney Goodman, a research associate at Cal Tech. He is an expert in the area of rule-based neural nets, and I know that I can use this product in a lot of other contexts. The reason is this: There are masses of data out there. NASA collects data. So does JPL, and there is so much data stacked up in storehouses. How do you extract any information from these masses of data? Well, one way to do it is by using rule-based neural nets carefully, and Goodman developed a technique for doing that. So we jumped in with both feet and learned how to use this approach.

Our first project came from Bell and Howell. It involved analyzing the scintillation phenomenon in the rear spring projector. What they wanted to do was to minimize the scintillation. That's a scattering theory problem.

We did life cycle costing for the State Architect's office. We did some things for Chevron which were very interesting. There again, we developed a tool for the client. Chevron wanted to detect the occurrence of high fluid pressure when a well was being drilled. They found that frequently there would be a huge discontinuity in the pressure when a certain level was reached. It would blow up the drilling equipment, and they wanted us to model why that pressure built up. If they knew the mechanism that contributed to the high pressures, then they'd be able to predict when it could happen. That involved solving some partial differential equations. Genesis was the model that we finally settled on from there.

We did a number of projects for General Dynamics, but they came to an end in 1989. General Dynamics is a defense firm. Their government contracts dried up with the end of the Cold War. All of those guys moved out of our valley. The ones who remained with the company are in Arizona now, and there isn't much in the way of defense work. So we're having to retool everything that we do now not to be so dependent upon places like Hughes and Northrop and General Dynamics which isn't even around anymore.

Next in the list are in-house sponsored projects which unfortunately we had to take. I don't like in-house projects, but if you have got students who are registered already and you have trouble coming up with the right kinds of projects for them, then you've got to do something because ours is a required course. My problem is that I have to find at least three projects because we have three options in our department. We have a computer science option, a probability/statistics/operation research (OR) option, and we have an applied math option. So we need an OR problem, a computer related problem, and something that involves scientific computing like a partial differential equations problem. Now I can't always find them. I spend a lot of time looking around. I'm generally successful, but not always. Sometimes it just fails, and then we look for something in-house.

There's an interesting story I want to relate about our work for HFC Software. That is a little software house in Santa Monica. I noticed that one of our students, a sophomore, had a summer job working for them. This company developed packages for the Macintosh that produce interesting looking diagrams and textured pictures of various kinds. It was almost like a game. You can create various geometrical objects and then you can texturize with certain things. I guess they wanted to make a commercial product out of that. So they hired a Harvey Mudd sophomore, Ben Weiss, a very bright young man, who was adept at programming and very good in mathematics. At the end of his sophomore year, Ben was hired by HFC for one summer. They made him a job offer almost immediately. That's how much they loved him. It was $75K then to stay on. He continued with them for a year, during which time he developed the guts of their program. Ben was the man who developed this HFC package called Kaiser Power Tools. Kaiser was the name of the owner of the company. Fortunately Ben decided to come back to Harvey Mudd to get his degree. When he came back, I said, "Gee, this is a great opportunity. I need a computer related project, so why don't I just get HFC, which loves Ben, to give us a project while Ben's still here. He's got to take the Clinic. What better situation could we hope for? It's just ideal." So, sure enough, we got the project, and I made Ben the team leader. That was the logical thing to do although it meant that he went from 75,000 to 0 dollars. That's right. He went to zero, but that was required by the rules of the Clinic. But wait a minute. There is a danger here that you have to watch out for. The liaison in this case was the boss. (HFC is a small software business, remember.) The boss is not really a computer programmer, so when he came to talk to us, I tried to get him to give us a problem that was well defined. As I mentioned before, I like to have a stopping point. I like to know how much is enough because that tells me that we're done. I've had bad experiences in the past when people keep coming back and asking for more things all the time. But I could never get Kaiser to tell us anything other than "Go and do something imaginative." What kind of a directive is that?

Now I didn't like this. Everything told me to avoid this project, but, on the other hand, I had Ben Weiss, and HFC loved Ben. And Ben was so creative, how could we fail? As it turned out, Ben was creative, but the other three people on the project and their faculty advisor, who was absolutely first rate, never did measure up as far as Kaiser was concerned. Ben was great but the other guys were lousy. What the boss wanted to do was to come in, interview everyone, and hand-pick all the people who were to work on his project. I didn't allow him to do that. We can't. And so he was always critical. No matter what we did, it was never enough. It was not creative enough. It was not *imaginative* enough. We did an enormous amount of work, and what Kaiser eventually ended up with was beautiful, but the poor guy never was happy. He paid us, but he made his discomfort known. So there's a danger of getting somebody as a liaison for whom everything seems to be just made to order but if the liaison is quirky, you're going to have problems. Anyway, that was HFC Software. By the way, Ben went on to work for them when he graduated with a 6 figure salary and stock options.

We did a couple of projects for Hughes Aircraft, and we'll be doing more. These were Claremont Graduate University (CGU) projects at the Information Sciences Institute. We did a number of projects for JPL. Some of those were with CGU, which has been very good about supporting us.

Here's another project I want to say something about. Los Angeles County Superior Court called the wife of one of our faculty members to jury duty. So there she was, the way we all are, disgusted at wasting her time, sitting around and waiting to be impaneled. When she got home, she talked to her husband who got the idea that there should be a Clinic project to take care of this problem by devising some technique to impanel juries with 99% confidence or so. He wrote up a proposal to the court which decided to accept it. We did the project, which involved a nice queuing theory problem. Students came up with a result that would save the LA Superior Court $300,000 a year, with just a slight modification of what court officials were doing. They turned in a report and, as you might expect, nothing happened. This occurs a lot. You do the work, it's funded, but nobody does anything with it. Now most of us who are in applied mathematics realize that that's an option.

This particular faculty member wasn't happy with the lack of response on the part of the Court. So he went to Channel 5 News. It turned out that the anchor of Channel 5 News at that time was running for the LA Board of Supervisors, and, sure enough, the anchor said, "Here's another example of waste in government; this study proves that you can save $300,000 a year and they refuse to do anything about it." So they sent the whole anchor team out to campus and they videotaped everybody involved in the project. Then the executive officer of the L.A. Superior Court threatened to sue me, the faculty member who had gone to Channel 5, the dean, the president of the college and the chairman of the Board of Trustees. Why? Because the Court had paid for this study and therefore it had a confidentiality agreement, right? That is in the agreement that I send out to everybody. So we're going to keep our mouths shut about this because it's theirs. We broke the agreement and the executive officer was going to sue us. What do you think happened after that? Well, the president talked to the faculty member, and the faculty member refused to do anything about it, citing grounds of academic freedom. So we had more visits from Channel 5 News, more things on the evening news, and it was getting mighty hot. Then suddenly everything stopped. We figured that something happened. Maybe there was some accommodation made, or some understanding arrived at.

And so, thank God, it got settled, but this is something you have to be aware of. If you involve people who are not applied mathematicians in this program, they should be made aware of the fact that the results of their work may be totally ignored. That's one option that your client has. It's his product after all. We signed over all rights to what we did, and we can't tell the client what to do with it.

When I got my Ph.D. at Berkeley, I went down the peninsula to Palo Alto to work for one of the aerospace firms there. The group in which I was employed was designing communications satellites that they would launch. There was an antenna that had to be kept pointing toward the center of the earth. Well, as we all know, the gravitational field is a conservative field, so if there is no damping, once there's any kind of a vibration in a satellite, it will tend to oscillate. That couldn't be allowed to happen. They had to damp those oscillations. The way they did it was to passively damp out the oscillations, so that when the satellite got up there, they would unfurl a band of steel at the end of which were springs and a weight. By using these attachments, some of the vibrations of the satellite could be damped. How did they determine the constants for the springs and dampers? By running simulations on a Philco 2000 machine that they had: months and months of simulations to determine the right values for these constants. So what did I do? My job was to get the best possible values for the constants. I was very proud of my results when I told my boss, but he was not happy with me at all. I couldn't figure out for a while why my boss was unhappy with the results I had. Then I found out that the company was selling time to the government on its Philco 2000. So, by running the Philco 2000 machine for months at a time doing just parameter studies, they were raking in an awful lot of money. They're in business, right? So they want to maximize their earnings. Why should they take the advice of some kid just out of graduate school? They ignored what I had to say. I should have known at that time that it was not necessarily having the right results that mattered because there are lots of other things to optimize.

Here's a war story about the Jet Propulsion Lab (JPL). One of the projects we had with them was to work with a software package they had developed. Understand that this was in the days before $\TeX$. JPL

Figure 2. Team leaders' Meeting at HMC

had developed something called the General Purpose Photographic System (GPPS). It was the precursor of TEX. They had tried to develop this huge package in-house twice but it was not free of glitches, and so they gave us the package with instructions to make it work. I should have known better. But I said, "Let's have a stopping point here; how will we know that it works?" I was given what looked like a very good stopping criterion. JPL said, "When we can put in a line of text and get it to come out the other side in the clear, then you're done." That sounded pretty reasonable to me. I was a little leery, but I thought we could do what JPL wanted. But it turned out that we couldn't. It was a God-awful mess. Students were unhappy. The problem was that nobody likes to work on other people's software. I mean, nobody. So I had a lot of trouble in keeping the students' interest up. In the end, we couldn't get the code to come out. We kept trying to patch it up, but it wouldn't come out. I had to hire people for the entire summer. It went all the way into next semester, and finally we got that one line of text to come out. At that point, I said, "That's it." And we sent it back to JPL. Now I just avoid any project that looks as if it involves fixing up someone else's software. We'd much rather write our own or do something from scratch or use patented software. Anything but someone else's software that has to be fixed. There's a lot of stuff like that out there, by the way, if you want it, but I would recommend staying away from such things.

Here's a different kind of war story. We had a client who pulled out at the very last minute. It was Rand Corporation, as a matter of fact, and their problem involved chaos, applied to war game theory. Everything looked as if it was going to work out, but at the very end the funding dried up. This happens a lot. That's why, when I'm developing new projects, I like to have half again as many projects as I think I'm going to need. So, for example, if I need four, I think I'd better get six, because a couple of them are going to dry up for one reason or another. Maybe the company will move to Arizona in September. I don't know what will happen, but I know that we always lose some. Well, that's what happened to this one. In this case, it was too late: The team had already been formed. The Clinic is a required course, remember, so we ran with it, anyway, and did the best we could.

One of my own projects involved Teledyne Microelectronics which manufactures hybrid microcircuits. These are circuits that are folded on top of themselves. A complex kind of methodology is used when you create these things. Among other things, they are designed in such a way that all the heat producing elements are on the top. Now when you run the circuit, these heat-producing elements, of course, will send heat into the interior, but they also heat up themselves, and if they get too hot, they will cut out and the circuit fails. So what the company wanted to do was to examine a hybrid microcircuit which has multiple layers. They wanted to design the thermal properties of each layer in such a way that they could be sure that the heat that would be reflected back to the top would be minimized. At least we wanted to guarantee that the life of this hybrid microcircuit would be longer than a certain minimum amount of time. It was an interesting problem which involved solving the heat equation, of course, but with the discontinuities which arose because of the various layers. With the heat equation, if you have any kind of discontinuity at all, the convergence time of the usual algorithms is quite slow. In addition, we had the three dimensional problem, and that was just miserable. So we searched and searched for a way to handle this problem, and then I remembered a friend of mine who worked in multi-grid methods, which is a technique of varying the mesh size in a clever way so that you can actually speed up the convergence of the algorithms by using multi-grid software. But we had to learn all that stuff, and, boy, was I worried coming to the end of the project that nothing would work. With the software that we had before, nothing was converging. Finally, multi-grid did it for us. That was how we produced a tool for Teledyne. What had Teledyne done before this? The same thing that Allied Signal had done. They had gone out and built these things and then they bench tested them, and when they blew up or they didn't work, they went back and changed the thermal parameters of the layers. They kept iterating this until things finally got to be more or less the way they wanted. So the tool that we developed for them did the simulation. This saved them a great deal of time, and it's still being used, so thank God for multi-grid method.

That's a lot of what mathematicians do. We do mathematical modeling and produce tools for engineers to use when they actually do design. There are two types of projects, and I want to emphasize the difference between them. One is an engineering design project which is what the engineers do, and the other is a mathematical modeling project. Here's the difference. In an ideal engineering design project, somebody gives the engineers a design objective: build this widget that operates under these conditions. Or optimize this under certain given conditions. So you take those design objectives, and you determine the specifications and operating characteristics of the environment, and then you identify whatever technologies you have to use in the design process. Then you construct a prototype in the lab to see if it works. If it does, then the design meets the client's needs, and you deliver it as is. Usually it doesn't work, so you keep tinkering with it until finally you get something that *does* work, and you give that to the client. That's the engineering design process. What we do in mathematics is somewhat different. We have some kind of a phenomenon that we need to model. We need to create a mathematical model which we experiment with. So here is the phenomenon, you prepare it for modeling, you produce a model, you solve it for certain special cases, interpret the results to see if it does what you want it to do, and then you try to validate it. In certain special cases, you get data from some source. If you can validate it, then it's ready as a predictive tool, and you deliver that to the client. If it's not ready, you go back and you circle enough times around until you get something that is good enough. Now that's the kind of projects I like. The students like the modeling projects, too, because they are open-ended. You start with some kind of vague statement and try to produce a model that describes what is going on there.

Now our colleagues in engineering are sometimes short of design projects, and they will take modeling projects, and, on occasion, we will be short of modeling projects, so we take the design projects. The difference in what we do is not absolute. But the modeling projects are my favorite because I'd rather have something that the students can get excited about.

Let me add one more word about the reports. The report in the Appendix was done for ESRI which is a geographic information systems (GIS) company. What is a GIS package? It's a package for collecting, managing and analyzing geographic and demographic information. The product that ESRI sells is basically a software package that has within it the entire road network of the world. So you can tell where the roads are anywhere: all the nodes and all the edges, all the geographic information is there. As a client of theirs, you use a programming language within their software package to design a delivery schedule, for example, one that visits certain nodes in a minimal amount of time, or, to cite another example, to locate a business in some optimal place. The language extracts that kind of information from the package.

This little company started ten years ago with 7 people. They now have 700 employees and they do about $300,000,000 worth of business a year, and they are growing at a very fast clip, especially in Eastern

Europe. That whole region is opening up, and a lot of businesses are moving in. Every time there is a MacDonald's or a Jack-in-the-Box, somebody has used a GIS package to figure out where to put those new franchises.

If you want to visit a number of nodes on the graph–say it's a weighted graph–then the algorithm used is a Dykstra algorithm. Unfortunately, the complexity of the algorithm increases very fast. When you have a huge graph like one which contains all the roads in the U.S., it becomes computationally prohibitive to use Dykstra's algorithm. So what they asked us to do was to find an algorithm which was sub-optimal but still good enough and quick enough for their clients to use. That's what we did. We developed an algorithm called the AELSGU which our students actually coded and inserted into their package called ArcInfo. That was a mess because ArcInfo is a huge package. The documentation for ArcInfo takes an entire shelf. So our students had to learn how to navigate in ArcInfo, how to design their algorithm, and then how to install it within their package. This is what they did, and they did a very good job.

One last example: ThreeCom Corporation is located in Santa Clara. The company manufactures software and hardware to support computer networks. They've been growing very fast by buying up other companies, thereby acquiring a huge range of software that they have to maintain. So they have this big inventory of software. Now obviously they don't want to make copies of all these things just to have them sit on the shelf waiting for somebody to order them, so they asked us to design an optimal inventory system that would satisfy their customers. What does it mean to satisfy their customers? Well, what the team eventually did was to look at the past performance of how these things had sold and they extracted information from that to predict what should be done.

When I formed the team, I had only one Harvey Mudd student on it, a young woman by the name of Dana Evans. I also had a couple of graduate students who were not totally committed to the project. They were being supported by the Clinic because that's how we use a lot of our income: to support graduate students. The only other student on the team was a math/econ major, and she was not very strong, so a lot of the work fell on Dana's shoulders. Halfway through the project, my graduate students dropped out because they didn't want to pay the high tuition at CGU. They decided to register *in absentia* and therefore they didn't feel committed to stay with the project. Dana not only picked up the pieces, but she brought that project to a very successful conclusion. I had a phone call from the VP of Marketing at ThreeCom about three weeks after we wrapped up the project. He was absolutely delighted with what the team had done and the company decided to use what they produced. In fact, the program that we delivered performed much better than what they had before, and so not only were they going to use it, but they also gave us another project the next year.

Dana did this on her own. In fact, she got the prize in the department for outstanding performance in a math clinic project. I find that our women do a much better job at being team leaders, and the reason, I think, is that they can sense when problems are going to arise—personnel problems, for example, which always arise—and they tend to keep people together and move them along. And that's what Dana did. She just picked up the pieces and brought everybody back together to finish up that project. I've noticed this now several times. I don't know why I hadn't noticed this before, but it's happening so often now that I'm beginning to think that whenever I can, I'm going to have a woman as the team leader.

Project Deliverables II

Tom Davis
Milwaukee School of Engineering

At the Milwaukee School of Engineering (MSOE), where I worked for 28 years, we had a lot of projects along the lines of what Bob Borrelli did at Harvey Mudd. We probably did somewhere in the neighborhood of 1000 projects: a large number, and during my first two years at Super Steel, we probably did 20. One thing that I would encourage people interested in these projects to do is to identify up front the project variables. Hearing a little of Bob's horror story brought to mind the importance of defining, very clearly in writing, what the variables are and also what the milestones in the solution process might be. One of the things that we want to know is where we're at, right? Avoid the kinds of projects where the client asks you to develop a new process to test such and such a thing. Those are really very, very difficult to do.

At Super Steel we have been successful about 75% of the time, but the 25% failures contain the seeds of success if one learns from them. A very good friend who worked for many years at research had an attitude about failure. Every time his group failed in a research project, they'd have a party to celebrate the failure. Now this had to be an honest-to-god failure, not just giving up on something, but a genuine failure and the reason was that, on average, one out of every 500 of their projects would fail, so that when they failed, they took the attitude that their failure was behind them and they could move on to a string of successes.

Of course, some of what my friend considered failures would not be considered to be failures by other people. You may find that you have thrown a lot of resources in pursuing a path which turns out to be the wrong path, but even though you're closer to the right one, you can't back up and start over again because you're running out of time. So the definition of the deliverables is really a critical issue. It's also important to design up front the process that you're going to use to get to your result. In fact, I think there are some pretty good models around. People do a lot of these kinds of projects.

Legal Agreements

Let me say something about the non-disclosure agreement which Bob Borrelli alluded to and statements about confidentiality like the one used at HMC. I've been to a lot of companies that already have their own. My attitude is that that's *their* agreement; we're doing the project, so the client is going to sign *my* non-disclosure agreement. I've already paid an attorney to write this up, and I'm not going to go back every time to have my attorney vet somebody else's non-disclosure statement. My non-disclosure agreement is a few paragraphs long and in plain English, and some that I'm asked to sign are seven pages long and in legal-ese, and that makes life very difficult. In any case, just to be on the safe side, be sure to get everything in writing.

Inter-disciplinary Teams

Something I would encourage is teaming up with other people. Don't just use the math people. Work with the engineers. We're trying to get people ready to graduate. Right? When they graduate, we want them to be successful and make a six-digit salary the first year they're out. Now they are not going to be surrounded by a hundred other applied mathematicians. In fact, they may be very isolated in their work environment, surrounded by a hundred engineering types whom they have to support. So the faster you can get them used to working with other people, the better off you are. That's one of the things that I think works really well with some of the projects that we do.

Illustrative Projects

One of the projects we did that was a lot of fun concerned the Milwaukee County Stadium. The problem with the stadium is that it is one of the few major league stadiums that has a concrete outfield wall. There was a concern that every third or fourth game, players would be injured by running into the outfield wall. A salesman sold the Stadium some padding for the outfield wall on the basis of the difference in sound made when a bowling ball was dropped on a concrete floor (it made a thud), and the sound made when it was dropped on this foam rubber (it made a lot less noise). So people were persuaded that that was a good thing to put up. After they spent a lot of money to do this, they found that it was ineffective in preventing injuries. Then they came to us for help. Through accommodation simulation, we designed a four-layer composite cover that does things ergonomically. When you touch it, it feels nice and soft, but this ergonomic layer doesn't really absorb anything. That's what the other layers underneath do. You can hit the cover with your fist as hard as you want and you won't hurt your hand.

The good news is that the first night it was installed, somebody from Kansas City playing here went head first horizontally into the wall to catch the ball and got up. Immediately. After the game was over, they took him to do a CAT scan of his head and everything was OK. He wasn't injured.

The general manager of the Brewers at the time—this was in the 1980's—was Harry Dalton. I didn't watch the game, but I saw it on the news. The commentator said that this was the kind of concrete wall that made sense. So I called Harry Dalton and I said, "Hey, it works, right? We're getting all excited." And Dalton said, "Don't get so excited. All baseball players have hard heads."

That was a team-type project, and it worked out really well. A lot of arguments occur when people are learning how to work with one other, and sometimes those kinds of disputes help a lot.

We worked on some other projects like Lantern. That was during the first Iraq war, Desert Storm. Lantern was the military code name for the equipment that allowed jet fighters to see at night. Most of the electronics for that were built in Milwaukee under a subcontract from Martin-Marietta. The first problem in developing that project was that the printed circuit boards which took an entire Cray I computer to operate had to go on a single printed circuit board about as big as an average-sized book. This controlled everything that went on in the fighter. The second problem was that the thermal coefficients of expansion between the components of the board and the board itself don't match when a jet airplane dives from an altitude of 50,000 feet down to sea level, in a matter of a few seconds. What happens? If you do that often enough and the solder joints suffer from fatigue, then the circuit fails. So what we were asked to do was to identify a material that had similar thermal coefficients of expansion for the components of the board. Since the company that we were working for manufactured printed circuit-boards, we used a technique involving a laser-taken picture of the printed circuit board. The temperature of the printed circuit board is changed, another picture is taken and superimposed on the first so that you can see all the thermal stress areas. Now that's really nice. But what we needed to do was to print these images and in order to do that, we required somebody who really understood the mathematics involved. The good news is that all the components which had an estimated life of over 450 flying hours are still in use.

We only charge our clients for out-of-pocket expenses. We do not charge them for the time spent on their projects. Now there are probably good reasons to charge them for that, but our projects help us to develop good will in the Milwaukee area. MSOE, for example, is still a good friend of Super Steel, and friends are really important because you may end up with an unexpectedly large donation for the college all of a sudden. For example, we charged the Milwaukee Brewers $9,000 for a project after having done several other projects for them, and about 6 months later the Milwaukee Brewer Organization donated $100,000 to the college. It did this because of the good will that our projects had engendered, not necessarily because we did a really good job. In fact, the largest single donor in the history of the school is a company that we did projects for. MSOE is a private college so donations are especially important. We're talking long-term endeavors here, and I got to be personally very close to a lot of companies. I ended up on 14 of their boards of directors.

We worked on the Trek bicycle which has a platinum frame. There was a cohesion problem with the paint, and as a result of the work they did on the project, our students got a dollar apiece and their names on a patent. How many students graduate from college with no industrial experience to speak of, but with a patent to their credit? And it was really interesting when Trek came by to give everybody their dollar. They brought sweatshirts for everybody as well. We had something like that happen 20 times or so, and, of course, we have had less of a problem with students finding a job.

I'm sold on the value of this sort of activity. In fact, I want to see student projects for virtually every course past the sophomore level, regardless what it is. If you have to write a term paper, you are better off writing a paper for somebody who's going to use it rather than writing a paper as what might be called an academic exercise. An academic exercise is, by definition, one that has very little or no practical value. I hate those kinds of things. So the goal that I always had in my classes was to have every student work on a real-world project for a client and for free. A really great job might lead to summer employment at the client's, but this was not necessarily the case. What was important was the work done in class by the students on the project.

I forget the numbers now, but once I calculated that millions of hours are wasted every year, hours that could be put to better use in a classroom by helping a client company do what it wants to do. If only we could find a way to turn that into practicality. Developing friendly relations with industry is really how you get donations for your institution. In my view, there's just a tremendous number of hours that could be used much better than to have students work a couple of math problems unrelated to reality. Instead, it would be better to say, "I have a problem for you, a real one."

Here's a problem that I turned over to a class. This company made grain elevators, the ones you use on the farm, not the one's that you see along the side of the road. There was a real problem with raising and lowering these elevators, with restrictions on how far they could move. Well, I ended up making this a class project with a week to work on it. The company received 25 written reports from a math class on how to raise and lower grain elevators. They picked one they liked. From the company's point of view, it was the best, but there were a lot of good solutions.

We used to do projects for Allis Chalmers in Milwaukee before that company went under. Once, I remember, the company needed a mechanism to get rid of rocks for a new combine that it had built. Every time the combine encountered a rock, it would rip the insides apart. Allis Chalmers got all kinds of solutions from their engineering staff, but they were too expensive to implement. Each of them cost anywhere from $300 to $1,000 in production, and the company couldn't afford those prices. They were at the end of their rope, and at the point where they were going to go out and hire somebody to look at the project. Instead they came to our creative thinking class with this problem. The students spent a week working on the project and produced a solution that actually ended up costing $9.95.

Those are the kinds of really neat things that can happen, and those are a few of my war stories. Now, at Super Steel, I'm on the other end, and you may ask what we can offer academia.

Let me give you an example of a recent project. I don't know if you've ever heard of E Coat, an electro-deposition paint. That's really neat stuff because the application happens under water, and the procedure uses electricity. The paint actually penetrates the metal. Most of the cars built today are coated to prevent the body from rusting, and you may have noticed that cars don't rust out like they used to. E Coat is rust resistant. The problem here had to do with the painting cycle. All parts had to be painted, some deep-coated, other parts not, but all on one line. Mathematically, how do we optimize this? There is an algorithm to optimize the through-put insofar as it relates to the production scheduled for a period of time like a week. But what's the optimal way to line up the parts so that you can paint in multiple colors, including the E-coating and everything else at the same time? That was a good project. The E Coat facility that we had was a $3 million machine that took 9 people to run. How do we incorporate that machine in an optimal way? Those are fun things to work on.

Siegel: Let me come in here to ask a question of Bob Borrelli. Do you have a tracking procedure to follow up on people within the industry once you make a contract? One of my problems is that there's such a flux going on within companies today. Do you lose sight of the people you deal with?

Borrelli: No, because we make contact at the highest level that we can, and those jobs don't change very often. The VP of Marketing whom we deal with at ThreeCom, for example, is going to stay there a while. Other people may come and go, but I talk to him first, and then, when he becomes aware of a project, he will put me in contact with other people. That's a stable situation, and the same thing is true of Allied Signal. I start at the top. Other people will come and go, but he's there so I don't have to keep track of everybody. It helps a lot, as well, to try to keep a project on site at the company. Because then you get to meet a lot of people you may have to deal with. This permits you to walk down the hall and say to people as their path crosses yours, "John, I haven't seen you in a long time. I have a question for you." All of a sudden, the whole thing blossoms, and the project gains momentum.

Siegel: A follow up on this: Does ThreeCom have an internal mechanism for identifying potential projects and anticipating your coming to them? A lot of time is spent trying to find a project. When you're talking to people at the highest level, they don't always know what's out there, unless there's a corporate culture which allows them to do that.

Borrelli: There's a corporate culture that requires employees to communicate because of the way money flows. Anybody who wants to do anything down on the corporate ladder is dependent on the boss' having a budget in order to release money to get the job done. So the boss generally knows what's going on.

In any case, it always takes some talking. So we go to the client, and we talk, and eventually something comes out of the conversation. I normally like to get an ordered list of projects, with just a paragraph of description for each of them. Then we think about the list and talk among ourselves, and then we reorder the list. We bounce things back and forth until the procedure converges. Ideally we want to do something that we think we'll be good at and something that the client thinks that it wants. I like to get about three project ideas for each client. That works out pretty well.

Davis: One good source of projects is your Development Office. It makes contact with a lot of companies. We put together a little seminar for our development staff to acquaint them with some of the services that we could provide potential clients whether they employ our graduates or donate money. This is a service which we recycle every year, and we tell the Development Office by the 15th of July that if it has some ideas for projects that require students, then here's the form. You write out a paragraph, you do this, you do that, and you send it to Professor X. We took what we would get from the Development Office and wound up with a list of maybe 100 projects every year. That's not just for mathematics; we circulated it throughout the whole school. "Do you have a class that you could do this in?" we would ask. "This one might take a week, and you could do it right in class," or "Is this one going to take the whole quarter? Is this going to be a senior project?" Finally we had probably 75–80% of them accepted by somebody.

What I have found is that, unless you get particular individuals on a particular day when they consider what they have to be a major problem, then they might not make the connection. They need to hear about student projects over and over again. That's really difficult. I even find myself in industry having fallen into this rut: Looking at a problem, trying to get it solved, when I could just have called up MSOE.

Let me make one final comment about arranging projects at the highest level of a company. In the past 6 months, I've been involved in a couple of meetings with our president and the CEO of another company. They each told their subordinates to work something out, and it just didn't gel because nobody came to the party prepared. I have better luck in getting people like foremen and students together. There are, of course, a zillion ways to do things, and different ways work with different companies in different settings.

Siegel: I think you need to know your neighborhood and decide again whether you want to stay there. I think you have to make yourself aware of what's going on. I find that I read the business section of the local paper much more than I used to, just to see where the companies are, who is moving in, and what sorts of operations are in the area. I think that you need to stay abreast of changes in the corporate scene.

Borrelli: I think there's a difference, too, if you're looking for a funded project, then that's a little more difficult. You can talk to a lot of people with ideas for projects, but they may not have the funding to hire you.

At the time **Tom Davis** wrote this chapter, he was working at a Milwaukee-based company called Super Steel. Previously he served as Dean of Faculty at the Milwaukee School of Engineering, and consequently his background includes both academic and industrial experience, providing him with a unique perspective on student consultancy work. The chapter concludes with a brief dialog among some of the workshop participants who have been introduced previously.

Using Projects from Industry to Teach Mathematics and Statistics to Liberal Arts Majors

Daniel Maki
Indiana University

Introduction

Since 1990, Indiana University has offered a junior-senior level course which is designed to teach selected students how to use mathematical and statistical techniques to solve real problems and projects presented to them by companies and by units of government. In this chapter we discuss the nature of the course, the types of projects, the mathematical and statistical topics usually taught in the course, and the results of evaluations of the course.

Background and Motivation

The course described in this chapter is part of the Indiana University (IU) LAMP program. The acronym LAMP stands for Liberal Arts and Management Program, and the associated program is designed to provide selected liberal arts majors with tools to help prepare them for a career in business or government. The LAMP program started in 1990 with classes of 25 liberal arts majors per year and it now enrolls about 100 students per year. Students typically enroll in the program at the start of their second year on campus, and most have majors that are not in the sciences or any traditional quantitative field. Planning for the program included a special course to provide these students with quantitative tools that were not normally part of their degree requirements but which would be valuable in seeking and holding a job in industry. Two IU faculty members, myself, Daniel Maki, in the Mathematics Department of the College of Arts and Sciences and Wayne Winston in The Kelley School of Business, proposed a course in which the key focus would be student teams working with an industry client on a project which would require the students to learn and apply mathematical and statistical techniques. It was also planned that the students would make both oral and written presentations to their clients, and the clients would come to campus for the final oral presentations. The course, as just described, has now been taught every year since the academic year 1990–91, and the basic plan has been followed quite closely.

The motivation for the course was the belief that students in majors such as English, history, Japanese, and fine arts, would not be likely to benefit greatly from another standard mathematics or statistics course. Instead, they needed to learn a variety of techniques and, even more important, they needed strong motivation to learn and apply the techniques. This motivation was to come from being on a team that met with a client and then worked to solve a problem posed by the client. The material covered in class would be related to the goals of the project and the students would be asked to carry out activities which showed that they understood the material and that they could relate it to their project. A typical example is the following:

The Credit Union Scheduling Problem

The Setting: (This information is given to the students when they first learn about the project. They will next meet with the client to ask questions and to learn more about the setting.) The manager of the local credit union branch has asked for help in scheduling part-time tellers. The problem is that demand for tellers varies greatly depending on the time of day, the day of the week, the day of the month, and the month of the year. If there are too many tellers, then some are idle for long periods of time (at a cost to the credit union). If there are too few, then customers wait a long time in a queue and are unhappy (at a cost to the credit union).

The Task: work with the manager of the credit union to develop a satisfactory method of scheduling part-time tellers, so that goals of the manager are achieved.

The next steps: after meeting with the credit union manager and talking within the team, it was decided that in order to develop this scheduling method, the following activities would need to be accomplished:

1. Determine the current pattern of arrivals at the credit union and develop a method to forecast arrival by day and time of day. The credit union has transaction records that will help greatly in developing the forecasting methods.
2. Collect data on service times so that a queuing model can be developed. This required that team members visit the branch and record service times.
3. Develop a queuing model that relates the number of tellers in service to the average length of the queue and the average time a customer spends in the branch.
4. Work with the manager to specify goals for the time an average customer should spend in line.
5. Develop a formula for the number of tellers needed to achieve the manager's goal for the average time a customer should spend in line.
6. Develop a scheduling algorithm for part-time tellers that accommodates the requirements for part-time work (for example, a shift may need to be at least 3 hours long) and that achieves the goals of the manager at a minimum cost.

Reports and Presentations

It is not enough for a student team to do a good job of studying and "solving" the problem presented to it. Team members also need to convince others that their work is worthwhile and that, in some sense, it accomplishes the task given to the team. This usually involves writing a report that describes and defends the work completed by the team. It also involves an oral presentation to a client. The audience at that presentation may well include people who have not been involved in earlier discussions, and it is critical that the terminology and notation be clear to all. If part of the audience does not have a technical background—for example, management—this must be considered in designing the presentation. Some of the key points that need to be considered in the report are as follows:

1. **What is the problem or area of study?** This may seem obvious to the team studying the problem, but it may not be obvious to someone who is now presented with the results of the study. It is very important to set the scene carefully, so that the results of the study will make sense to any interested party who reads or listens to a presentation about it.
2. **How was the study carried out?** Were data collected by the team or provided by someone else? Was statistical sampling used, and, if so, how was the sample selected. How were the data used? Was the study an analytical one or was it a simulation or some parts of each?

3. **What are the results of the study?** The team needs to describe what it has learned about the area of study and its solution to any problems posed. This is, of course, a very critical step in the process, and more than one method of presentation may be needed. For example, graphs and charts may help to show results, and it may be useful to compare the results obtained in the current study with those obtained in earlier studies or by different methods in the same study.
4. **What are the conclusions of the study and how valid are they?** This part of the report or presentation is the one where the results of the study are compared to the real world, and they are interpreted in the context of the problem posed at the start of the study. As part of this comparison, the team needs to look seriously at its work and note the limitations (if any) of the work. This is also the place to note the options for additional study on related problems that may be of interest to the clients.

The role of class lectures and laboratories

Most of our students have not worked with the mathematical and statistical tools that they will need to use to carry out their team projects. Thus each year we need to teach them this material which they then can apply in a practical setting. However all of this must be done fairly quickly so that work on the projects can begin, and, as a result, there is not much time for extra material to expand on the topics covered. Our approach has been to alternate classes between a lecture/discussion format and the use of a computer lab. Except for some preliminary work on the basics of using the spreadsheet Excel for mathematical and statistical work, every topic discussed in class is related to at least one of the team projects. All teams learn the basics of each topic. We assign some homework on the class material and there is one exam. The exam is a practical one, emphasizing the use of the concepts developed in class. One day a week we meet in a regular classroom and we cover a new topic, *e.g.* forecasting by using multiple regression. The second day of the week we meet in a computer lab where the goal is to learn how to apply the tool studied in the previous session. Some topics, of course, require more than one week of lecture and lab to complete. In a typical week, we often need to forecast for some of the projects. For example, in the Credit Union problem discussed above, the team needed to forecast the number of customers who would arrive on a given day at a given hour. In class, we covered multiple regression methods, and we worked with data sets and regression in the lab. These methods eventually worked very well for the team. As expected, it took some work to find an optimal set of variables to use in the forecast. Some of the important variables turned out to be the categorical variables of day-of-the-week, month-of-the-year, date-in-the-month, day-before-holiday, and payday.

The exam for this course takes place about three quarters of the way into the semester and it is held in a computer lab. It involves the same sort of problems that were covered in the computer labs, including the topics we know that the students will need to understand for their projects. Students who do not do well on the exam must learn the material they missed and present it to the instructors to show that they now understand it.

Following the exam, the class lectures stop and the entire focus of the course is on the team projects. Each team meets regularly with its course instructor and reports on progress to date. The team must produce a draft report and must do practice presentations as the day of presentations draws near.

The types of projects used in our course

Our experience indicates that certain types of projects work best with our students. These are projects that involve mathematical and statistical tools that we can teach rather quickly in class and for which we can provide good examples and good exercises for our students. These projects typically involve a subset of the topics of forecasting, quality control, inventory control, queuing, simulation, scheduling, and routing.

We avoid projects that are primarily programming tasks and those that involve a substantial amount of data entry. Of course, most projects involve some data entry, so the critical question is the amount of data that is already in electronic form and how much must be entered by the students. We also avoid projects that require a large number of student visits to the client. A few visits are almost always necessary and, in fact, it is very useful to have the students meet the client at the client's home base. However all of our students have a full course load, and it is often difficult for the team to arrange a number of visits to a client. Our goal is to have at most 4 or 5 such visits and in many cases only 2 or 3 visits are needed. The projects we obtain are quite varied in the degree of structure provided by the client. Some could easily be called unstructured, while others are very structured. Unstructured projects are ones where the key assumptions are not given to the modeling team. Instead, the student team must collect and use data and study the basic situation before making the key assumptions and forming a model to consider. Of course, this also involves talking with the client to be sure that the team is on the right track and moving in a direction that will be useful to the client. For example, a team might be told that the client wants to set up a warehouse in a location that will help them reduce delivery time. However, they are not initially told much about the options available or the constraints on the new site. The team must ask questions, collect data, make assumptions about the needed capacity, and then study alternative locations based on alternative goals. In general, of course, the company will play an important role in defining the study and helping guide the team. We did one such project where the team came up with two possible locations, the company chose one of them, built there, and it seems to have worked out very well for them.

Many client-driven projects are structured ones that come to a student team from a company or a governmental unit and the project is presented as a problem that must be solved. The student team must work with the client to understand the setting and to figure out which aspects of the problem are the important ones and which ones can be ignored or minimized in solving the problem. For example, the problem might be one in routing and scheduling for delivery trucks. Key facts might be that the company owns four trucks of a certain sort and that the maximum number of hours the drivers are permitted to drive per day is ten. On the other hand it might not be important that two of the trucks have Cummins engines instead of Ford engines. In such a situation, the team, serving as consultants to the company, must work with company officers to learn the key goals of the project. Are they simply to minimize delivery costs or are there more complicated issues, perhaps involving customer wishes and company growth plans?

The mathematical topics usually used in the projects

Of course, the exact topics covered will vary from year to year as the projects vary. However, after a few years, we learned that some topics came up regularly and we could expect to cover them almost every year. For example, we begin each year with a basic discussion of statistics and with the statistical and data manipulation capabilities of Excel. This takes only a week or two, but it is essential to most projects. We also cover forecasting almost every year. Although not all projects will need to do forecasting, at least a few of them will need to do it. We teach regression and some basic time series essentially every year, and we have the whole class do examples and homework requiring some forecasting with "real" data sets.

After basic statistics and forecasting, there are no topics that we cover every year, but there are several that we often cover, depending on the projects. The most common are simulation, queues, scheduling, routing, inventory control, and quality control. In each case, we cover only the basics in class, with the understanding that an individual team may need to learn more than the rest of the class because its project requires that. Most of these topics can be implemented using Excel, but, in a few cases, the students also need to work with a software package such as SPSS, SAS, or Maple. Also, for some teams, it is helpful to the clients to present them with tools for implementing the results and recommendations of the student team. In such cases, we have often worked with the team to use the language VBA to make a user-friendly implementation.

How to find good projects

We have not yet discovered a foolproof method of finding projects, but we have learned which methods work well and which ones do not usually work. First we note a method that is a natural one to try, but which did not work very well for us. Namely the method of writing to the CEO's of many companies in our area and asking them to look for projects. If you do not know the CEO and if you are not introduced by someone who does know the CEO, then you are unlikely to get a response in a timely manner. Occasionally this method will eventually yield a good project, but, in general, it did not work well for us. The best method for us has been to directly call someone at a company, someone whom we either knew or had been introduced to by a mutual acquaintance. Although the person contacted might not be the one we eventually worked with, he or she usually knew the correct person to contact and was usually willing to make the contact for us. We have found that it is very important to mention concrete examples of projects we had already completed or believed we could do with our students. It is important to quickly convince a potential client that you can provide this person with a service, and the best way to do this is to cite past successes.

In addition to industrial clients, we have had great success in getting good projects from government agencies, especially those involved in health and welfare. We note, in particular, our long relationship with the Indiana State Department of Health. We have worked with the people there almost every year, and they have been a great client. They have a wealth of good data, they have many interesting questions to ask, they are very busy, and they do not have the time to do everything they would like to do for forecasting and analysis of their data sets. We have also worked with hospitals (on inventory control) and a county government (the best way to use limited road repair funds). Again, in these cases, it is important to know someone who can introduce you.

One word of warning about finding and working with new clients: it is very important to discuss two logistic items with the client before agreeing to do a project with them. The first is that there must be a liaison person at the company or governmental unit, someone who agrees to answer students' questions in a timely fashion. In return, of course, you must work with the students to be sure that the students do not expect the client to end up doing the project. In general, we ask that students limit contact with the client to essential items and, if in doubt, talk with the faculty supervisor before contacting the client. The second item is to determine the status of the data needed for the project. We have had cases where the client believed and told us that a great deal of good data was available in electronic form, and they would provide it to the students. However when the time came to provide the data, it either was not available or it was in very bad shape with lots of holes and wrong entries. A good rule of thumb is to ask the client to "show me the data."

Evaluations

We have carried out several methods of evaluation of this project-based course, including standard class surveys and having a doctoral student write a thesis that included interviewing many students in the course. The picture is a very favorable one. Students seem to appreciate the format of the class, they like meeting clients who represent real companies and governmental units, and they like the sense of satisfaction that comes from finishing a major project. Some of them complain that they have trouble working within the structure of the team or that the workload is not carried equally by all teammates, but this is not a common complaint. Perhaps the strongest praise for the class comes from students who have been out of school for a few years. Many of these students report that they have used their project as a building block for job interviews and for an important part of their resumé. Also, several of these students now have jobs with consulting firms, and they have often returned to speak to the current class and to recruit for their firm.

One additional form of evaluation is that given by our clients. Almost all our clients have asked to participate again, and we have worked with many of them two or more times.

Summary

In summary, our experience using real-world projects from industry and government to teach mathematics and statistics to liberal arts students has worked well. We have now used the same format for 15 years, and both students and faculty believe it has accomplished the goals we set for the course in 1990.

Daniel Maki works at the main campus of a large state university which includes an urban campus in the state capital as well as six regional campuses. His article deals with a student subpopulation not affected by the consultancy projects described by the other contributors to this book. What can mathematics departments offer the non-major?

Mathematical Modeling in ICIC Projects

Robert Fraga
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All the participants at the Marquette workshop who have contributed to this book come from large and/or elite schools. Can those of us who work at small liberal arts colleges which don't have the prestige of Harvey Mudd College do what Bob Borrelli does there? How can we start building the track record which he suggests is necessary to win projects? These are not easy questions, and answers will vary from one place to another, but I can share my experiences at Baker University, where I have worked for the past eight years and at Ripon College, where I worked at the time of the workshop, in the hope that these may suggest ways to go about doing math consultancy projects.

First a hypothetical question, "Why bother with projects like these? Isn't life in academia hectic enough without taking on what could be a punishing additional workload?" There is evidence that consultancy work has real value for undergraduate students. Such work exhibits the relevance of mathematics to contemporary life, and it develops skills that are useful in a broad spectrum of activities: problem solving, learning to work as a team, meeting deadlines, and communicating effectively. As I wrote in the introduction to this collection, students who participate in a consultancy project will not ask "What good is mathematics?" after their experience in that project. And the project involves more than math: frequently there are social, legal, and political issues which arise during the work. Examples of some of these have already been given. Borrelli's tale of the L.A. Superior Court project is one. I believe that students at a liberal arts college benefit from this kind of exposure, in part because it helps them to realize that there can be non-mathematical consequences when they bring analytic skills to bear on a problem.

Given that you want to try this, how do you get started? Again, as I have written above, this will depend largely on your institution and environment, but I can indicate what we did, first at Ripon, then at Baker.

I worked at Ripon for the academic year 1996–97 before moving to Baker, and there was time to do only one project that year. In a sense, my students had already begun that project at the time of the Marquette workshop. They had toured the Ripon Community Printers (RCP) plant and knew about the company's problem of scheduling jobs on a weekly basis. Because the company used FoxPro, my students had to use that as well to produce a jobs schedule given a week's workload. There are issues common to all projects, regardless of the institution where the work is done and for whom it is done. One of those issues is that you are constrained to use the client's software package or you have to persuade the client to go out and buy something else. The first option is generally the one which the client prefers because it involves no extra cost.

Here is another common issue: projects are done primarily for their educational value. I believe that all the academic speakers at the Marquette workshop made this comment, and it is one to which I shall return in a bit. With RCP, this was not a problem.The company has intimate ties to the college, and it was willing to allow students to work on a problem of real importance to it. RCP actually used what my students produced. The students made a couple of presentations, one at the 1997 spring meeting of the

MAA Wisconsin Section and another at the college for colleagues and RCP staff. As a result of their MAA presentation, the students were invited to write a brief paper on their work for the COMAP Journal, a reference to which is given in a footnote to the chapter on COMAP.

We worked pretty much for free on the project, and this is something which many of you getting started on consultancy projects will have to do as well. RCP paid for the reference books which my students bought for the project, and the company paid for a return air ticket for one student in the team. This student graduated at mid-year and accepted a job with Intel where he was working at the time of the company presentation in Ripon. I agree with George Corliss that students should not be out of pocket as a result of their participation in a project, but until your school has the reputation of a Harvey Mudd or a Towson, it is unlikely that any money will exchange hands in contracting students to work on a company problem.

The environment in northeast Kansas was at first a good deal less hospitable than the environment I had left behind in Wisconsin. There, following a suggestion of Bob Borrelli's, we had set up a board of advisors for consultancy projects, and this board helped to land a number of workshop participants projects for their students to work on. There was nothing of the sort in northeast Kansas, a region of the country which, for our purposes, was synonymous with Kansas City and its suburbs. I did what Martha Siegel recommended and started digging around to make the kinds of contact that would lead to projects. This resulted, through the assistance of a Baker alum, with my coming to know the Initiative for a Competitive Inner City (ICIC). It was through ICIC that I scored several of my projects over the years at Baker.

The ICIC is the brainchild of Michael Porter, a professor at the Harvard School of Business. Porter contends that the inner city has business potential which is not fully recognized or exploited by private entrepreneurs, and the ICIC exists to promote the exploitation of that potential by companies interested in locating in the inner city. With funding from the Federal government and institutions like the Kaufmann Foundation, the ICIC was able to set up urban laboratories for its philosophy in four US cities, one of which was Kansas City. The agency through which the ICIC worked there was called Kansas City Advisors. Kansas City Advisors coordinated assistance offered by groups of MBA students to companies already established in the inner city which had a promising track record. That program offered us a way to get involved in the work of the ICIC and find consultancy projects for Baker math and computer science students.

Our first project under ICIC auspices was a collaborative effort between a team of my students and a team of Executive MBA students from the University of Missouri in Kansas City (UMKC). The project involved a business survey of the renovated Jazz District of Kansas City. This neighborhood, which centers on the intersection of 18th and Vine Streets, now houses the Kansas City Jazz Museum, the Negro League Baseball Museum, and an old cinema, the Gem Theater, which has been restored as a conference center and auditorium (see Figure 1).

Our specific task was to assess the parking demand in the Jazz District, with emphasis placed on those times which were seen to be critical. A commonly accepted benchmark for parking analysis, the one which the students used, is the capacity necessary to satisfy demand 85% of the time. The strategy adopted by the team which worked on this problem was to divide demand into two categories: 1. Noise: the parking demand generated by people who worked in the District or went there to visit the museums. 2. Spikes: the demand generated by the special events scheduled at one of the facilities in the District.

A computer profile of demand was created for each of six time intervals, seven days, and two seasons (summer and "non-summer"). Using the benchmark figure of 85%, the team recommended that a 300-car parking facility was required, principally to meet demand on Saturday nights in the summer.

The report done by the EMBA students, which included the parking analysis done by my students, was warmly received by the Development Corporation for the Jazz District which agreed to implement the recommendation to create a parking lot in the District. This project was one of four field studies accepted for presentation at an ICIC conference in Boston the following year (1999). It tied for first place

Figure 1. The Gem Theater

with the Wharton School of the University of Pennsylvania in that competition. A judge for the contest indicated that the quantitative work which had been done by the Baker students had been impressive, and, in my opinion, distinguished this project from the other three field studies in the competition. The ultimate compliment bestowed on my students' work, of course, was the decision to implement the report's recommendation.

Over the next three years, teams of Baker students worked on two other ICIC consultancy projects. On both of those projects, we flew solo after having established a track record of sorts with the Jazz District project. The first of these two projects involved a cleaning company in Kansas City, concerned about turnover in its part-time work force. To mitigate this problem, the company wanted to know how many full-time workers it would need to employ in order to satisfy its contractual obligations. Because of the nature of the work, done between 5 PM and early morning, and the fact that many of its workers did not own cars, the company was prepared to lease a fleet of vans to move its workers from job site to job site. Our task was two-fold: to determine the number of workers required by the company, subject to certain legal and company-imposed constraints, and to produce van schedules to transport the workers around the city at night.

This was a difficult problem which the students were not able to solve entirely although they made real headway with it. Determining the number of workers required by the company was handled, in part, as a linear programming problem and the van scheduling was solved by critical path method. By the time our month ran out (more about this later), we had determined job assignments for workers but had not sequenced them nor taken into consideration acceptable meal breaks in the cleaners' evening work. We needed to look more closely at assumptions about the van routes, as well, and this could have been done if the company had hired one or two of my students during the summer; but the company declined to do so and presumably still organizes its work shifts with felt marker and white board. As Bob Borrelli pointed out, your client is under no obligation to implement your suggestions or to continue the students' work beyond the time allotted.

The second of these ICIC projects was again neighborhood-related. The Hispanic Development Corporation (HDC) of KC had relocated in an old warehouse district just west of downtown. HDC wanted a traffic analysis of this neighborhood, called the West Bottoms, and the previous work by Baker students

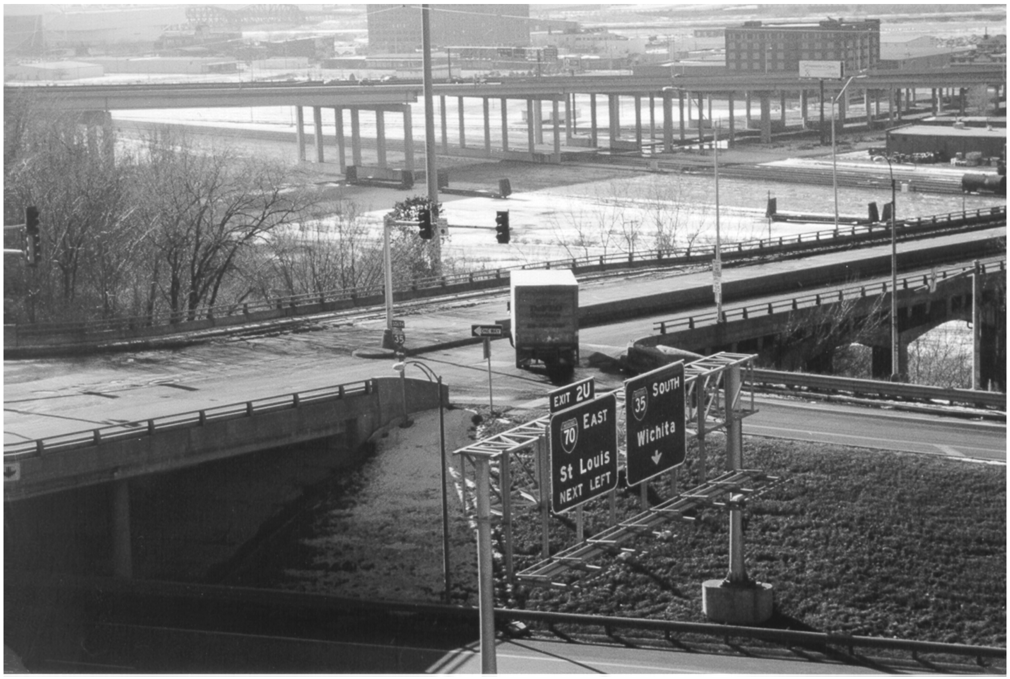

Figure 2. I-35 12th Street Viaduct Intersection

on parking in the Jazz District suggested that we were naturals for the job. Maybe this illustrates the importance of establishing a track record with the local business community, but there is a downside to that as well. You risk being type-cast, for example, as a traffic analyst, and find it harder to get projects which involve other problems and different kinds of math. The work on the West Bottoms fell into three subprojects, only two of which involved math that my students could do. In their report, my students cited the difficulties encountered by motorists in evading trains being shunted from track to track, sometimes across main traffic arteries in the West Bottoms, and illegally parked trucks which blocked public thoroughfares for hours and days at a time; but no attempt to do a quantitative analysis of these blockages was made, partly because it was too difficult to get data on the problem.

Of the remaining two subprojects, one involved accessing the 12th Street Viaduct from the I-35 exit ramp (see Figure 2) and the other involved traffic flow away from the Kemper Arena, a local stadium, after a sporting event there. This second problem is modeled by a partial differential equation, and a full analysis of it lay beyond the scope even of the more advanced students in the project. They contented themselves with determining, on the basis of data collected after a hockey match at the arena, whether a linear model realistically described traffic flow away from Kemper. The 12th Street Viaduct question, however, involved only first semester calculus, and that was handled by first year students in the project.

There are a couple of what George Corliss might call war stories associated with the calculus of the 12th Street Viaduct problem. We were interested in estimating the length of truck which could negotiate the turn from the I-35 exit ramp onto the viaduct as a function either of the angle of the intersection or of the width of the ramp. We had reason to believe, as I shall indicate below, that one or both of these could be altered. We wanted to know what the angle of intersection was currently, and since we did not have a theodolite, we fell back on a piece of string and the Law of Cosines. Taking measurements meant dodging the semis that came off the ramp onto the viaduct, and dodging those trucks meant leaping over the guardrail, in a kind of Kansas version of a Portuguese bullfight.

The 12th Street Viaduct is of historical importance because it is the first triple-decker roadway to have been built in this country. Over the years, it has undergone alterations, many of which are unfortunate; it has suffered wear and tear as well, and Kansas City was about to undertake a study to estimate the cost of restoring the structure when the student team at Baker did its work in January 2001. This led our team to invite an art historian to come talk to us about the history of the viaduct. If the city does decide to restore it, then altering the intersection of the viaduct and the I-35 exit ramp can be incorporated into the cost of restoration, and a fairly minor part of the cost, at that.

How did we find out what the traffic problems in the West Bottoms were? We did not go to the HDC for these but, instead, roamed the streets of the neighborhood for a day looking for ourselves and interviewing people whom we accosted in the street. Here's another war story for you: The first person we talked to was a man who had just been released from prison (the West Bottoms contains a correctional institution), and he told us that he had no idea what traffic problems the district had! It was a truck driver who alerted us to the hazards of leaving the I-35 exit ramp to go west into the West Bottoms along the 12th Street Viaduct. As frivolous as these anecdotes may seem, they add spice to the project and will help students remember what they did years after the project was completed.

When were all these ICIC projects undertaken? In the month of January. That is the month when Baker has its Interterm, a time for our students to do something or to study something not normally done during the regular semesters at the school. A month is not really adequate for a consultancy project as speakers at the Marquette workshop have pointed out, and there is an educational cost to cramming so much in so little time. For one thing, the faculty advisor of the team needs to play a much more active role than would be the case at any of the schools represented by speakers at the workshop. This cuts down on the educational value of a consultancy project at Baker, but currently that seems unavoidable. You may find that a modest start like ours, which lasts only one month, will work best for you. Maybe this can be viewed as pump priming, necessary to build up a track record in order to persuade your home institution of the value of this kind of academic exercise. I should admit that projects always dribble into second semester, what with delays in the report preparation and presentations before the client and suitable audiences. Students have not objected to this extension of their work although naturally they are involved in other courses and activities in second semester and must squeeze in the time to tie up the loose ends on the consultancy project.

There are common themes in math consultancy projects. One of those, given above and cited by most of the participants in the Marquette workshop, is that these projects are undertaken principally for educational purposes, not as a way to provide cheap labor for industry or local government. In fact, one of my principles in accepting a project is that it not deprive a worker of a job. If the principle of "pedagogy first, industrial worth second" is not accepted, I would recommend that you not touch a project with the muddy end of a barge pole. Here's an example of a project I bailed on because it was clear to me that the potential client had no interest in the educational value of the project. A coffee producer in Kansas City appeared to have a problem with mathematical content: Each week the delivery routes for his company varied because of demand. The company wanted a way to create routes which optimized something, i.e., they minimized time spent making deliveries or distance travelled in order to save fuel. This looked to be a Traveling Salesman Problem, and since I was and remain addicted to coffee, it looked like a dream job. I had visions of complimentary coffee delivered daily to the math department by a grateful client. But the client was clearly on another wave length; he was interested in results, he said, and he had no time for consultations with students or with student presentations. He said at one point in our exchange of e-mails, and here I paraphrase: "I'm in a hurry. I'm building an empire." The bit about empire is a literal quote. Reluctantly, addicted to coffee though I be, I broke off contact with the client. You can't win them all.

Here's another of the common themes of math consultancy projects. They are useful in recruiting students. Baker, like other universities, is trying hard to recruit students, and recently it looked as if our department was going to be given a quota of students to recruit. This seemed at first blush as if we were being asked to do the work of the Office of Admissions, and I was grumpy about any quota-filling notions which came from administrative sources, but let's not lose sight of the forest for the trees here.

Most colleges and universities are in competition for students these days, and anything which makes your program more attractive to students is to be cherished. Consultancy projects make your department stand out, and *they* are to be cherished. Yes, they do involve a lot of extra work, and, yes, it's worth it.

Like Martha Siegel, I find that I tend to carry the torch for consultancy projects in my department although I have managed to interest some of my colleagues in doing what I do. Ours is a department which combines several disciplines, including computer science, and one of my computer science colleagues is now promoting consultancy work and internships. I do not take all the credit for this; my colleague would have done these things without encouragement from me. Here is yet another example of how a small school can start a program like the ones discussed in this collection without too much preliminary ground work. Baker is located in a small town called Baldwin City. A few years back, when town-gown relations were none too cordial, I went to the office of the recently-appointed city manager to introduce myself. During this courtesy visit, I mentioned that, should the city find that it needed help with computer-related issues, the department would be happy to offer assistance. It took a while for this idea to germinate, but finally my visit paid off. Years later, Baldwin circulated a questionnaire about the services which it provided, and the city needed help in assessing the questionnaires which had been returned. This ultimately became the subject of a senior project for one of our computer science majors, an internship rather than a team project, but an example nonetheless of a "real world problem" which became the subject of an academic study. My colleague has done a number of in-house projects, and a group of his students has also worked on a problem for a company in Lawrence that deals with computer security issues.

Not all my work at Baker has been done under ICIC auspices. To escape being type-cast as a traffic engineer, I sought out a DNA analysis project with the assistance of a leading cancer research institute in Kansas City. It was difficult to "retool" for this, but the result was satisfying. A group of students working during an Interterm project with me and my colleague in computer science accessed a recently published data set consisting of 101 mitochondrial DNA sequences and constructed a set of phylogenetic trees composed of the subjects from whom the sequences were taken. Another project which a group of my students at Baker undertook arose out of a chance encounter with the contractor for the Wildcat Cafe, a student center on campus. This contractor had school addition projects in Liberty, Missouri, and my students did a critical path analysis of the construction processes involved in them. They also analyzed crashing costs for the projects for each of which there was a penalty if construction was not completed by an agreed-upon date.

As I have written above, I do not allow students to be used as cheap labor. None of the projects that we have undertaken cost employees of the client company their jobs. Frequently what we have done is to provide a company with the solution of a problem which would simply have gone unaddressed if we had not come along at the right time. Business trends of "downsizing" and "outsourcing," which were common phenomena of the 1990's and which remain with us, make this an opportune time for students to undertake consultancy work. Companies are often too understaffed to tackle certain kinds of issues that affect their operations.

Nonetheless, there is a fine line to be drawn here. Agreed, consultancy work is, first and foremost, an educational tool. This is *not* a way for academic institutions to turn a quick buck. On the other hand, clients do get their value for money from the projects that students work on, so student consultancies are not a disguised form of charity.

I contend that it is possible for small institutions like ones at which I have worked to support student consultancies on their own. But there is nothing to prohibit collaboration between universities and colleges with limited resources to make it possible for students to reap the excitement and personal satisfaction offered by consultancy work.

The First Example: Converting Part-Time Workers to Full-Time Employment

Statement of Problem

Woodley Building Maintenance (WBM) is a company in Kansas City, Missouri, that provides cleaning services for approximately 120 businesses in the greater Kansas City area. It has about 400 employees, approximately 100 of whom are currently full-time, the rest part-time. To reduce worker turn-over, WBM wanted to convert more of its part-time workers to full-time status. Many of the employees do not have cars and must use public transportation, or walk, to get to their job sites. The public transportation system in Kansas City shuts down around 11:00 PM, and a full-time worker at WBM will be on the job later than that. In order to provide transportation for its workers, WBM was willing to lease a fleet of vans to transport the workers from job site to job site.

Our task was three-fold:

1. Locate one or more Drop Off/Pick Up (DOPU) points.
2. Maximize the number of full-time workers required by WBM's contractual obligations subject to constraints which reflect company culture and legal considerations.
3. Produce a van schedule to meet the transportation requirements of the workers.

WBM provided a listing of job sites and the zip code areas in which its work force lived. Since the roster of job sites for which WBM had contractual obligations changed fairly rapidly, the strategy outlined in the following section needed to be flexible enough to take into account the changes in job sites in the company roster. Staffing changes also occur, of course, but it was assumed that the distribution of workers' residences in terms of zip code areas was constant.

Strategy

The density of the workers' places of residence determined the location of the DOPU's. The vast majority of WBM's work force lived in a rectangle approximately 4 miles by 9 miles located just east of downtown Kansas City, and extending south toward the I-435 bypass. The principal road which runs through this rectangle from north to south is Prospect Avenue along which Bus Route #71 runs. Considerations in locating the DOPU's included ease of parking for those few workers who had cars and proximity to a bus line workers could use in getting to their first work sites.

A visual inspection of the entire length of Prospect Avenue was made, and a decision with regard to the location of the DOPU's made as a result of that inspection. It was decided that the few workers who lived outside the rectangle alluded to above would probably have their own means of transport and would therefore be less dependent on the van services of the company.

Maps of all Kansas City bus routes were obtained, but after detailed examination, it was decided that buses would be useful only in transporting workers to their first job assignment and would not be used by workers in the course of their work shifts. Thus transportation between job sites and between the last job site for each worker and the DOPU's would be provided by the WBM fleet of vans.

Assigning workers to job sites was treated as a linear programming problem. Since the goal of the client was to convert as many workers to full-time status as possible, the objective function of the linear programming problem was effectively the number of such full-time employees. The constraints for the problem reflected company culture and legal considerations. The output of this part of our model was a set of values for the variables t_{il}, which is the time spent by worker i at site l.

The values for t_{il} were the input for the van assignment part of the project. This part of the problem was tackled by a critical path method (CPM) implemented on the software package we chose (see below). The network associated with this part of the problem consisted of a succession of work arcs W and driving arcs D. The set of earliest event times (ES) for each of the driving arcs formed the input for the

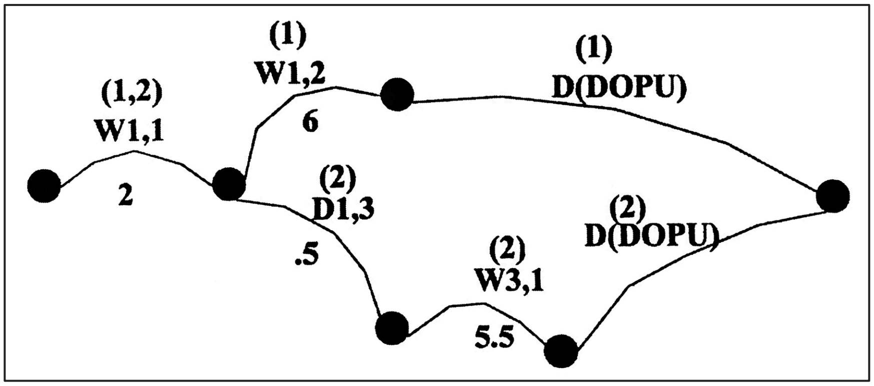

Figure 3. A portion of the drive/work network

van schedules. A sketch of part of the network is given in Figure 3. Nodes joined by a W arc represent the initiation and termination of work at that particular job site done by a particular worker or workers. Thus the subscripts in the notation W_{ij} indicate the job site i and the sequence position j of the work shift done there, e.g., $W_{1,2}$ is the second shift at work site 1. These arcs are weighted by the number of hours spent on the shift indicated by the second subscript. The drive arcs D were weighted by the estimated driving time between the sites (nodes) which they join. A default value of 0.5 (half an hour) was used in this model for a variety of reasons. This was a conservative estimate of driving time between any two sites, given the constraints on where the cleaners could work. The numbers in parentheses (i, j) indicate that workers i and j are involved in the shift listed below the parentheses, e.g., (1, 2) above $W_{1,1}$ means that workers 1 and 2 work the first shift at job site 1.

The set of earliest event times ES for each of the driving arcs D constituted the van schedules for each of the subdistricts in the model.

Implementation

The linear programming package used to optimize the number of full-time workers was “SuperLingo.” This software package has the capacity to handle 500 variables and 1000 constraints. However, after considerable experimentation, it was decided that even this package was inadequate because the number of constraints required by the WBM problem exceeded the amount available to the package. In order to overcome this deficiency without purchasing a more expensive upgrade of SuperLingo, it was necessary to divide the metro Kansas City region into three subregions. This was done by a visual inspection of a map containing all of the WBM job sites and dividing the region into three smaller subregions, each containing approximately the same number of jobs. The software package was then capable of processing each of the smaller work regions individually.

By entering the job sites and the number of man-hours needed to complete the work in each subregion, SuperLingo was able to determine the job sites to which each worker should be assigned and the number of hours spent there in order to maximize the number of full-time workers.

From the spreadsheet in which these assignments appear, a network model was created to aid in the determination of the van schedules. Determining these schedules used the Critical Path Method (CPM).

Results

The DOPU’s were located by visual inspection along Prospect Avenue. These sites were close to parking lots, where those workers with cars could leave their vehicles, and to bus stops for the buses needed by workers to get to their first job sites.

District 2	SITES																
WORKERS	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	HOURS
1		2	2		0.5												4.5
2	2.5					3											5.5
3																	0
4				8													8
5				8													8
6							5	1	2								8
7							1										1
8														3.5			3.5
9										4	1	1.5		1.5			8
10													2		3	3	8
11														8			8
12														8			8
MANHOUR	2.5	2	2	16	0.5	3	6	1	2	4	1	1.5	2	21	3	3	

Figure 4. SuperLingo Output for Subregion #2 in Kansas City

Job assignments were made by running SuperLingo on a linear programming problem. These (unsequenced) assignments were formatted as spreadsheets in order to be easily read.

Figure 4 represents subregion #2 of metropolitan Kansas City. Job sites are represented as columns.Since it was assumed that we would not distinguish among workers, numerical designations were also given to workers. The numerical value in row i and column j represents the number of hours worker i is assigned at job site j. Each row sum equals the total number of hours the worker represented by that row is assigned. This implies, for example, that full-time workers should be assigned eight (or in some cases seven) hours. Part-time workers were assigned between four and seven hours a night. The sum of a column equals the total number of man-hours needed to complete the job at the column job site. Some hand manipulation of the SuperLingo output was done to compensate for the program's inability to deal with certain constraints, e.g., those used to equalize the amount of time spent at any job site by the workers assigned to it. This

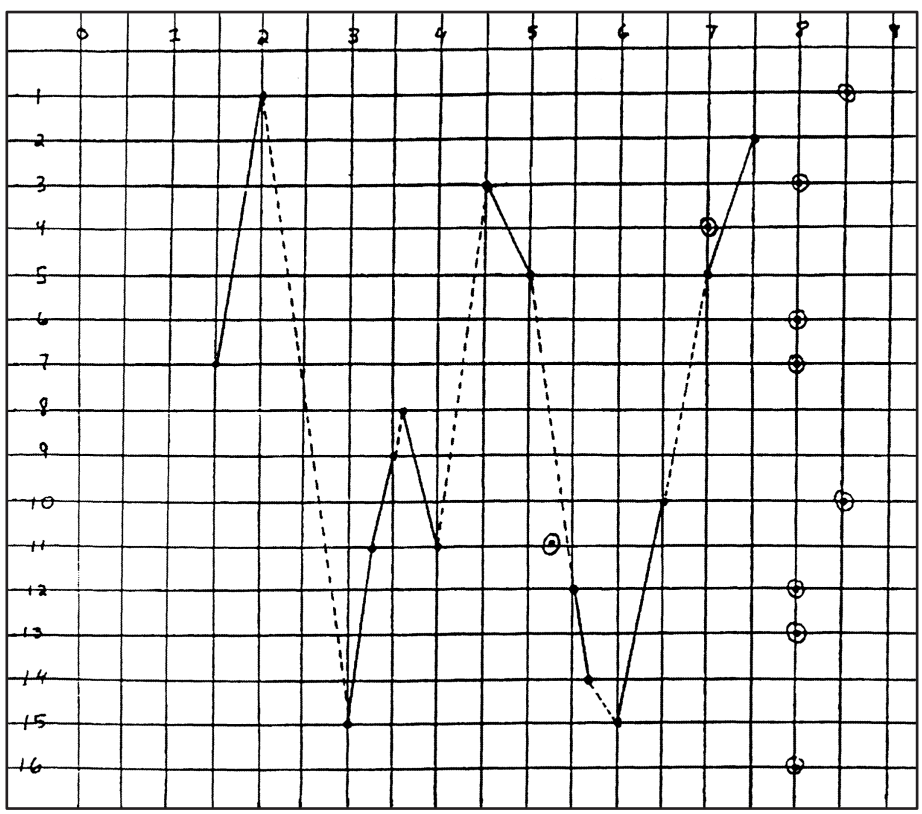

Figure 5. Van connections for Subregion #1 in Kansas City

manipulation was done with an eye to minimizing the van driving required to implement a solution to the WBM problem.

The relevant output of the CPM was SuperLingo's calculation of the earliest event times (ES) for the driving arcs in the network used for this part of the problem. Again, to preserve confidentiality, the job sites were identified numerically with an identification key supplied the client in a separate document. As in the determination of the job assignments, time $t = 0$ corresponded to 5:30 PM, $t = 1$ to 6:30 PM, etc. Figure 5 indicates the van connections for Subregion #2 of Kansas City used in the model.

The row numbers indicate job sites, the column numbers indicate times, with $t = 0$ again corresponding to 5:30 PM. Line segments which go left to right from site i to site j indicate that a van must drive from site i to site j at the indicated time. Dotted lines indicate that no workers are transported on those legs of the commute.

A Second Example: Assessing Parking Demand in the Renovated Jazz Districk of Kansas City

Description of the District

The District encompasses approximately eight city blocks (see Figure 6). The center of this district is the intersection of 18th and Vine Streets from which it derives its name. Currently in this district there are several major buildings. These house the Kansas City Jazz Museum, the Negro League Baseball Museum, the Gem Theatre, and the Gregg Fitness Center. Capacities for these facilities were supplied. Although the Black Archives was then located at 20th and Vine outside the district boundaries and the region modeled, it was anticipated that it would be relocated in a facility to the east of the Gregg Center some time in the following 16 months. So the Black Archives was included in the model developed in this project. These facilities are identified by labels in the map. Their hours of operation were determined and an estimate of the way special events held at these facilities would contribute to parking demand in the District was made as described below.

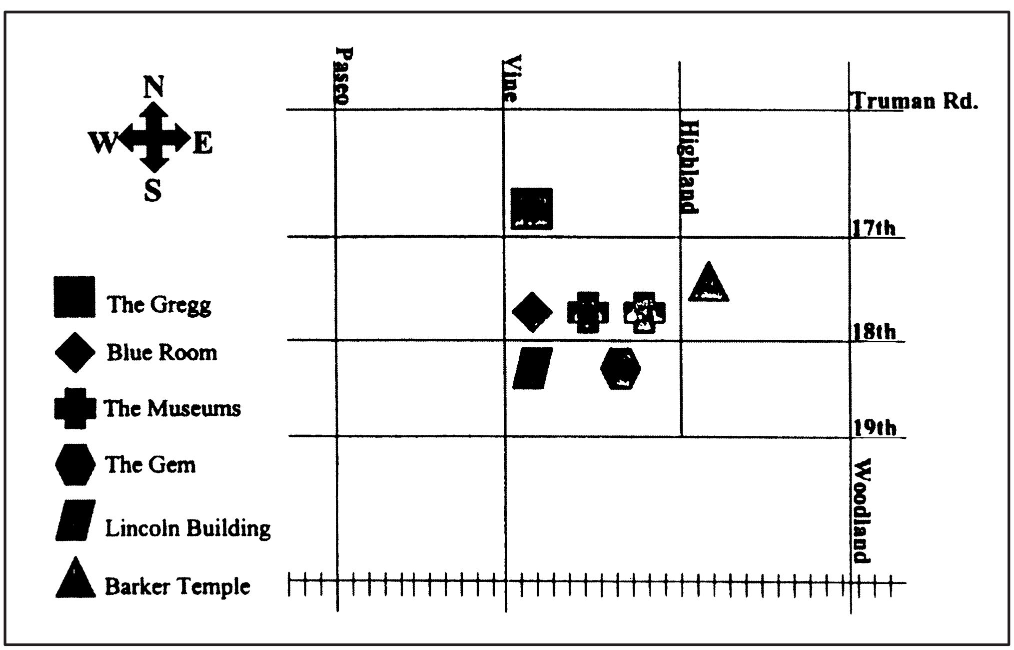

Figure 6. Map of the 18th & Vine Renovated Jazz District

Statement of the Problem and Strategy Development

Our client, the Development Corporation for the Jazz District, wanted to assess parking demand for the district and the number of additional parking slots required to satisfy that demand.

Two types of activities were considered: noise and spikes. Noise consists of constant parking demand generated by the District. This includes employees and customers whose numbers were assumed to be constant throughout the business day. Spikes consist of special activities scheduled in the district, such as tournaments, concerts, receptions, and lectures.

In analyzing parking, we decided on the following time periods for the model: 8 to 11 AM, 11 AM to 1 PM, 1 to 5 PM, 5 to 7:30 PM, 7:30 to 11 PM, and 11 PM to 1 AM These periods were selected to correspond with the opening and closing times of major businesses, as well as starting and ending times of special events. The model considered two seasons, summer and non-summer, since the demand in the late fall, winter, and early spring seemed to be about the same. Overall, this gave the model two seasons, seven days per week, and six time periods per day for a total of 84 time "frames."

Noise was assumed to be constant in each time frame. Spikes were entered into the model based on the frequency of special events recorded for the months of October, November, and December of 1997 and January of 1998. On the basis of these frequencies, the model employed a random number generator to determine whether an event took place at each of the major institutions in the District during a given frame. The answer to this question was either "yes" or "no." If the answer was "yes," the model produced the number of people who attended the event, added it to the existing total of people, and moved on to consider the next institution (or concluded a consideration of the frame). If the answer was "no," the model produced a "0" for the number of people involved and continued the process as described if the answer was "yes." Once the model determined the number of cars per frame required by people attending spike activities, this demand was added to the demand generated by the noise, and the number of cars per frame was calculated. This process was repeated for every frame, 10,000 times per frame, to produce an average distribution of parking demand in the District.

Implementation

To implement this strategy, a computer program was created to determine the number of cars in the District at any particular time.

Noise, in effect, consisted of the parking requirements of employees at the Lincoln Building, the Gregg Fitness Center, the Kansas City Jazz Museum, the Gem Theatre, the Negro League Baseball Museum, *The Call* newspaper, and the Black Archives. Noise data were gathered through a series of interviews with employees in the District. In addition to parking for employees, patrons for both the Kansas City Jazz and Negro League Baseball Museums were included in the noise category. Since the visitors to the museums created a steady demand for parking in the District, it was appropriate to include these figures in noise totals. The distribution of museum visitors was calculated by taking daily attendance figures and breaking them down, time frame by time frame, to accord with a typical daily attendance profile supplied by the museums. Figures for summer attendance were increased by 50%. Since the noise in the District consisted of employees and museum visitors and both of these groups were likely to create a fairly consistent demand on parking throughout the year, it was assumed that the level of noise was "locally" constant, i.e., for each of the model frames and each component of noise-related demand, there was a constant number of cars, subject to profile and business-hour considerations.

Spikes occur as the demand for parking created by special events in the District. The Gregg Fitness Center, the Kansas City Jazz Museum, the Gem Theater, the Black Archives, Barker Temple, the Blue Room, and the proposed outdoor band shell all contribute to this component of parking demand. For each of these locales, a number was calculated and entered into the program to represent the likelihood of there being a special event there.After the probability of a spike event's taking place was calculated, the number of attendees for each event was estimated on the basis of past attendance records. Sometimes this involved the use of a random number generator to produce a number of attendees from the (normal) distribution of attendance figures obtained from sources in the District. At other times, choices were made stochastically from the data received during our interviews.

Distributions and probabilities for each of the major players in the District were entered in the model code. We were then ready to simulate parking demand under a variety of scenarios. For each frame, a random number generator was used to choose a number between 0 and 1. The program then compared this number to the probability of an event taking place. Background noise was regarded as constant for each frame, so the probability of a contribution to parking demand from this source was 1 (100%). After a determination of whether or not an event took place, a random number generator was again used to calculate the number of attendees at the event. For an event whose attendance was normally distributed, a discretized version of the normal standard distribution was used. Attendance figures for major events at the Gem Theater were generated on the assumption that half the time these events involved 400 attendees and half the time they involved 500 attendees. This assumption was supported by data provided by the theater. Attendance numbers yielded figures for the numbers of cars involved by dividing by 2.2 [See Ref. # 10]. (Employee contribution to parking demand was calculated generally by assuming that there was one car per employee unless evidence indicated that employees arrived at work by public transport.)

The code, implemented on an IBM PC, made 10,000 runs per frame. For each of these, a decision was made whether or not events occurred, and in the case that an event did occur, how many people and cars were involved in it. An average number of cars was calculated (per frame) from the numbers generated by the 10,000 simulations. It was determined, for each frame and for the distribution of parking demand output, whether or not a given capacity was adequate to meet that demand. Values for the capacity parameter were: 0 (existing capacity), 100, 200, 300, 400, 500, 550, 600 (reflecting the addition of more parking slots in the District). The percentage of simulations for which the given capacity was adequate was then calculated. Results are given in the section below.

Two different kinds of run were made:

1. A simulation of current needs.
2. A simulation of projected future needs.

Current needs included the Band Shell which was projected to be in place for the summer of 1998 and the Black Archives relocation in the District, projected to occur in the following 16 months. Projected future needs included development anticipated over the following 24 months.

An attempt was made, for both sets of simulations, to highlight those times which were seen to be critical for parking demand in the District. Not surprisingly, these times were first the 7:30-to-11 PM time periods on Friday and Saturday, and second, the 5-to-7:30 PM periods on the same days.

Results

The results of simulating current needs were viewed as a calibration of the model. The model suggested that, with existing parking facilities, parking demand was satisfied only 75% of the time in the 7:30-to-11 PM time slot on Fridays and Saturdays in the non-summer months. For the summer months, this figure dropped to 61% for the 7:30-to-11 PM time frame on Saturdays. Also, during the summer months, the 5-to-7:30 PM period on Saturdays experienced only an 87% satisfaction rate. The addition of 100 car slots did not change the picture much because of the presence of a significant number of outliers in the distribution of demand generated by our program for the time periods considered (although the number of cars which could not be accommodated was reduced very significantly by the additional parking space). Still, the number of times when space was inadequate for the number of cars present was not really affected by this addition. The addition of 300 parking slots resulted in a satisfaction of demand throughout a non-summer week (with 94% on Fridays from 7:30 to 11 PM and 98% satisfaction for that same time slot on Saturdays).

For the summer months, these figures stayed at 94% for the Fridays and fell to 86% for the Saturdays. A 400-car facility provided universal satisfaction except for the 7:30-to-11 PM period on Saturdays when it fell to 92% during the summer months.

The critical times did not change for the projected demand which was simulated in a second run of the model. These were again the 5-to-7:30 PM and 7:30-to-11 PM periods on Fridays and Saturdays although the 7:30-to-11:00 PM period on Sundays yielded an 89% satisfaction rate for existing capacity which approached the benchmark value of 85% with which we worked. With the development envisaged for the Jazz District, the satisfaction rates dipped much lower than was the case for current conditions. For the 7:30-to-11 PM periods on Fridays and Saturdays, during the non-summer months, existing capacity met demand 42% of the time. During the summer months, this was projected to fall to 16% for the Saturday slot and remain about 42% for the Friday slot. The time frame which preceded this frame (5-to-7:30 PM) also illustrated how capacity failed to meet demand. On Friday, the satisfaction rate was 55% and on Saturday, it was 30%. Satisfaction rate also fell below 85% for the summer months on Sundays from 1 to 5 PM We observe here that only an additional 500-to-550 parking slots allowed the District to stay above the 85% benchmark for all frames in the model.

Acknowledgements

We gratefully acknowledge the support of the three MBA students with whom we worked on this second project: Eugene Agee, Cindi Dunn, and Gretchen Wells. These colleagues were role models for the undergraduates who worked on the project. On the WBM project, we acknowledge, with equal gratitude, the assistance provided by Po Lung Yu, Carl A. Scupin Distinguished Professor of Business at the University of Kansas, for his assistance in developing the strategy which was used. We also wish to thank Mark Wiley and Kevin Cunningham of Lindo Systems, Inc. for their assistance with the SuperLingo software package.

Let's wind up with a suggestion: Mathematicians interested in pursuing projects like those which Bob Borrelli, Martha Siegel, and George Corliss have described could contact them, or the schools at which they work, to explore in greater depth what is involved in student consultancy projects. This book has offered different models from which to choose, and what may be appropriate at one institution may be less appropriate at others. Our writers have all expressed a willingness to talk to those of you interested in doing at your home institutions what they do at theirs and to extend an invitation to visit them for special events, like Projects Day at HMC, or for extended stays of a semester or more. It needs to be said, however, that no special funds are available for such stays and you will probably have to arrange your own funding. Contact our writers for more specifics. Bob Borrelli has retired from HMC, and the Math Clinic is now directed by Michael Raugh. It would be advisable to address inquiries about attendance at Projects Day to him.

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Robert Fraga, the author of this chapter and the editor of this book, recently retired from Baker University, a small liberal arts college in northeast Kansas.

Appendix:
A Sample HMC Report

Final Report

"From Here to There: Finding a Short Path Quickly"

Fall 1995– Spring 1996

Prepared for

Environmental Systems Research Institute, Inc.

Clinic Team:

Tu Banh Rich Krabill

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Abstract

ESRI, Inc. of Redlands, California, is the leading maker of Geographic Information System (GIS) software that manages and analyzes data based on geographic information. One capability of ESRI's products is the determination of the shortest path between two locations in a road network. Dijkstra's Algorithm, currently being used to solve this problem, becomes computationally impractical when working with large networks. The Level Graph Search and Augmented Level Graph Search (ALGS) Algorithms can be used in these situations to greatly decrease the resources required for finding the solution, but at the cost of obtaining possibly sub-optimal solutions. The project involves coding and integrating the LGS and ALGS algorithms with Network Engine, ESRI's new programming library for developers of custom GIS applications.

Chapter 1
Introduction

This 1995 Mathematic Clinic project is sponsored by Environmental Systems Research Institute, Inc., the leading maker of Geographic Information Systems software. ESRI is located in Redlands, California.

1.1 What is a Geographic Information System?

Geographic Information Systems (GIS) are computerized tools that manage and analyze spatially-referenced data. This rapidly growing field has a wide range of applications, such as the planning of efficient routes for delivery or emergency vehicles, tracking of infectious diseases, and record-keeping of streets, pipelines, and other civil structures. In addition, GIS has an equally impressive list of commercial uses, such as the determination of effective locations for consumer outlets, improvement of marketing strategies from the analysis of geographic trends. The strength of GIS lies in its ability to give data a visual orientation, allowing one to make connections that otherwise cannot be observed with traditional charts or tables.

1.2 Profile of Environmental Systems Research Institute, Inc.

Founded in 1969 by Jack and Laura Dangermond, Environmental Systems Research Institute, Inc. began as a research group devoted to improving methods of handling geographic data. Today, the company is the leading producer of GIS software, capturing almost 29 % of the market and approximately $160 million in software revenue in 1995, according to the industry analysts at Daratech Inc. Currently, ESRI has more than 1,000 employees, 12 regional offices, 11 international offices and over 350 distributors worldwide.

1.3 ESRI's software products

ESRI's flagship product, ARC/INFO, is a high-end GIS which integrates cartographic capabilities with a relational database to provide a complete system for collecting, manipulating and visualizing geographic information. ARC/INFO managed all of the digitized map information that was used by the clinic team.

Arc Network is an extension of ARC/INFO that is used to model and analyze spatial networks, such as networks of roads, rivers, or pipelines. It has the capability to solve the shortest path problem as well as other optimization problems.

The Network Engine is a versatile programming library that provides the capability to define, store, traverse, and analyze networks. As it provides all basic network management functions as well as high-level analytic solvers that tackle specific problems, it will be used to build the next generation of ESRI's products. In particular, the clinic team is working on the solver that finds solutions to the shortest path problem.

Chapter 2
Project Definition

A common query made on networks of roads is the determination of the shortest path between two locations. This function, as currently implemented in ESRI's software, is not sufficiently time and space efficient when it is used on large networks of roads. In fact, all algorithms which solve the problem exactly (as Dijkstra's Algorithm does) are known to be inefficient. The goal of the clinic project is to find, code, and test other suitable algorithms to solve the shortest path problem more rapidly.

2.1 Shortest Path Problem Definition

The shortest path problem is a long standing and well-studied problem in mathematics (for a more complete background of this problem, see *Introduction to Algorithms* by Cormen, Leiserson, Rivest, 1990). Informally, the shortest path problem consists of finding a path between given source and destination locations on a map which minimizes the total distance travelled. More generally, the problem is to minimize a quantity called impedance (or cost) that is accrued while traversing the network of roads. In almost all cases, the impedance for any road will simply be the travel time, a quantity that is easily calculated from attributes of the road networks such as road lengths and speed limits.

Since roads in the real world might be one-way streets or might have different travel times depending on which way one is travelling, ARC/INFO gives each road two impedances, one for each side of the road. If a road is a one-way street, the impedance associated with that direction is set to a negative number.

The shortest path problem can be extended to allow for intermediate stops between the source and destination. An example of this might be to find the quickest path home from work stipulating that the grocery store and dry cleaner must be visited en route.

2.1.1 Mathematical Representation

The shortest path problem is modelled mathematically by a weighted graph, a collection of nodes and edges that connect the nodes (see Figure 1). In the case of a road network, each intersection of roads corresponds to a node in the graph, and each road segment (1 block long) corresponds to an edge. (In ARC/INFO terminology, an edge is also known as an arc.) The weight of each edge is simply the impedance. The problem then becomes finding the path with minimum total weight between the source and destination.

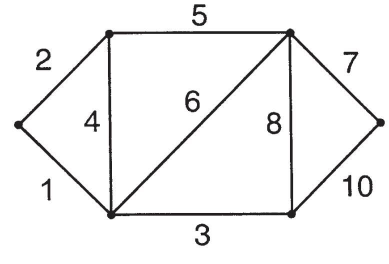

Figure 1. A sample network of roads represented as a graph, with nodes, edges and weights

Although the shortest path problem has traditionally consisted of finding the shortest path between two nodes in a graph, in the real world one rarely begins and ends a trip at intersections of roads. Therefore the shortest path problem will be slightly generalized to allow for the source and destination locations to be anywhere along an edge. In almost all cases, the impedance incurred for traversing a portion of an edge will be proportional to the length of that portion, expressed as a percentage of the edge length.

The use of graphs to represent networks helps to free the shortest path problem of any associations with particular networks such as networks of roads or rivers. By placing the problem in an abstract setting, only the most essential elements are preserved: the topology of connections between points in a system, and the impedances that in some way measure these connections. However, to improve readability and clarity, road networks will be used throughout this report instead of abstract graphs. Road networks are the most universal examples of graphs and they convey the problem statement of the shortest path problem most clearly. Despite the convention adopted in this paper, it should be remembered that the shortest path problem is equally applicable to networks of pipelines and even electrical circuits, for which the impedance might represent resistance.

2.2 Why "short" path?

The focus of the ESRI Mathematics Clinic Team is to find *approximate* solutions to the shortest path problem rather than the actual solution. The reason for this is that the shortest path problem requires finding the path with the minimum impedance, which can become too computationally intense in certain situations. In some cases it is more important to find an approximate solution to the short path problem quickly than to find the exact solution.

Typically, ESRI's customers work with networks that contain upwards of 100,000 nodes. The clinic team computed that the absolutely minimal data structure needed to represent all of this information in a computer occupies 100 bytes of memory per node. Therefore 50 Megabytes of memory are required to process a network with 500,000 nodes. Moreover, the data structures that are used by ARC/NETWORK contain much more information (such as street names, zip codes, etc.) than the estimates above, so that a network with 500,000 nodes can require over 200 Megabytes of memory. This is beyond the memory capacity of even many server-class machines. As a result, the computer is forced to swap portions of the network from storage into RAM as they become needed, a process that is relatively time consuming. The problem with Dijkstra's Algorithm is that it must examine every single node and edge in a network in order to guarantee that the solution produced is the shortest possible path. This implies that eventually the entire network must be swapped into the computer's memory in smaller pieces. As a result, Dijkstra's Algorithm performs poorly on large data sets since the time required for the swapping greatly exceeds the actual calculation time.

Moreover, this problem is not unique to Dijkstra's Algorithm, but is a fault of *every* algorithm that is designed to find the absolute shortest path between two points. The reason is that such a guarantee of minimized impedance is only valid if every possible path has been checked. As an illustration, suppose that the shortest path from Los Angeles to New York is desired, and imagine that a quick rocket flight from the west to the east coast was offered from a hard-to-get-to location on the west coast. Although it might take two days to get to the launch site, the resulting time savings would be considerable compared to a cross-country drive. If the roads leading to the launch site were skipped because of their inconvenience, then this solution would not have been found. Thus, every node and edge must be checked in order to guarantee that a path is optimal.

Consequently, every implementation of an algorithm that claims to solve the shortest path problem has the potential to spend most of its time swapping data between memory and virtual memory, resulting in unacceptably slow run times for large problems. The natural solution to this dilemma is to trade the guarantee of obtaining the shortest path for computational feasibility. Therefore, the clinic has been searching for algorithms that are able to find approximate solutions to the shortest path problem quickly by intelligently ignoring some of the nodes.

2.3 Company Profiles

The following company profiles provided to the clinic team by the support services at ESRI, are examples of how the finished product of the clinic project will be used in industry. The names of the companies have been excluded to protect their privacy.

One of ESRI's customers is a timber company in Scandinavia that uses Arc Network to find transportation routes for its timber. Every day the company solves approximately 2,500 different shortest path problems, most of which consist of finding the shortest route between a storage facility and a factory. Currently, the company uses a road network that contains about 430,000 nodes and 490,000 edges. Because of the large number of routes that must be calculated, a small savings in computation time for each problem will result in a large amount of time saved. Assuming that 2,500 routes are calculated in one day, a savings of just 1 second per route will amount to over 40 minutes of saved computation time!

Another company that could benefit from the clinic's work is a home furnishings distributor based in the United States. In the course of trying to find the most efficient way to link their distribution centers with their customers, this company also has to calculate a large number of different shortest paths. Each of the company's 20 distribution centers is the originating site for roughly 30 routes a day, each route servicing an average of 20 customers. At present, the company calculates the best routes by first finding the shortest path from each customer to all the other customers, then finding the best order in which to deliver the products. Our algorithm will reduce the time used for this phase of the company's operation and will allow the company to handle last minute additions and deletions to their delivery list.

ESRI has a customer in the United Kingdom who wants to find a short route between any two (or more) points in a network of 3.5 million edges (all the roads in Britain) in a reasonable amount of time. There is an additional stipulation that the route must stick to specific types of roads once it gets onto it, but the route may start and end on another type of road.

2.4 Project Goals

The three main goals of the mathematics clinic project are:

1. to find and compare algorithms that calculate short paths,
2. to code the algorithms to interface with Network Engine (see Section1.3), and
3. to test their performance, comparing both time and efficiency with the original algorithm.

In the first semester of the project, the clinic team formalized the project statement and began a comprehensive literature search for viable algorithms. The outcome of this search was the Level Graph Search Algorithm and Augmented Level Graph Search Algorithm (see Chapter 4). During the winter break between semesters, the clinic team attended training courses at ESRI's headquarters to become familiar with ARC/INFO. The second semester of the project consisted of the actual coding of the algorithms and testing.

Chapter 3
Dijkstra's Algorithm

Currently, Arc Network employs Dijkstra's Algorithm, the most well-known algorithm for solving the shortest path problem. This algorithm suffers from poor performance because it is required to examine every node in the network in the worst case. An example is given to illustrate this fact.

3.1 Dijkstra's Algorithm

The most well-known algorithm for finding the shortest path between two nodes in a weighted, directed graph was invented by Edsger Dijkstra in 1959 (Cormen, Leiserson, Rivest, pages 527–531). Dijkstra's Algorithm guarantees the optimality of its solution, so the algorithm must look at all the nodes in the network. In fact, it finds the shortest path between two nodes by finding the shortest path from a designated starting node to every other node in the network.

Because Dijkstra's Algorithm is currently used by Arc Network to solve the shortest path problem, the clinic team began the project by studying it to understand why it becomes computationally intensive for large networks of roads. As described in Section 2.2, heavy memory requirements coupled with the need to search through every node in the network cause Dijkstra's Algorithm to perform poorly.

3.2 An illustrative example

Dijkstra's Algorithm is known as a "greedy algorithm" because it looks at the best solution it has so far, and tries to extend it. Consider, for example, the graph in Figure 2. The source node is A, and the destination node is F. The numbers written along the edges are the weights (or impedances). At each stage in Dijkstra's Algorithm, the known minimum length from the source node to any node X is represented as $d(X)$. Initially, Dijkstra's Algorithm sets $d(A) = 0$ since A can be reached from itself with zero effort. The values of d for the other nodes are set to infinity, because at this stage it is not even known if these nodes can be reached from the starting node. This starting configuration is shown in Figure 2.

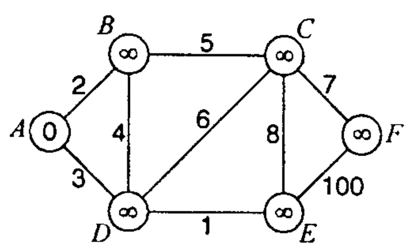

Figure 2. Initial state of a sample shortest-path problem

Dijkstra's Algorithm also maintains a list, S, of the nodes that have been examined. Initially, S contains none of the nodes. From the remaining nodes, the node with the smallest value of d is chosen. Once that node has been examined, it is added to S. For the first iteration (see Figure 3), node A is chosen since $d(A) = 0$. The value of d (the known minimum distance from the starting node) is updated for each node directly connected to A using the weight of the edge required to travel to it. In this case, both B and D are connected to A. Now $d(B)$ is updated to 2 since before B had a distance of infinity (see Figure 4). In

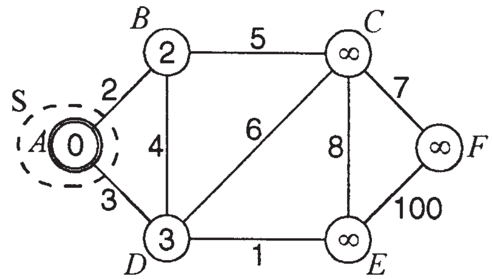

Figure 3. Node A is added to S and its neighbors are updated.

Figure 5, now $d(D) = 3$ since before D had a distance of infinity. Now that all the edges from A have been explored, it is added to the list S.

In the second iteration, node B is chosen from the collection of nodes not in the list S since it currently has the lowest value of d. For each node directly adjacent to B, the value $d(B)$ is compared with the sum of d for that node and the weight of the edge connecting it to B. If the latter is smaller, then the value of $d(B)$ is updated. This process of updating the known distances of the neighbors of a node is called "relaxation." In this case, node D could be reached in $2 + 4 = 6$ units of time by going through B, but $d(D)$ has already been assigned the value 3, which is less than 6, so $d(D)$ remains the same. On the other hand, node C has $d(C) = \infty$ which now gets changed to $2 + 5 = 7$. Now that all the edges from B have been explored, B is added to the list S (see Figure 4).

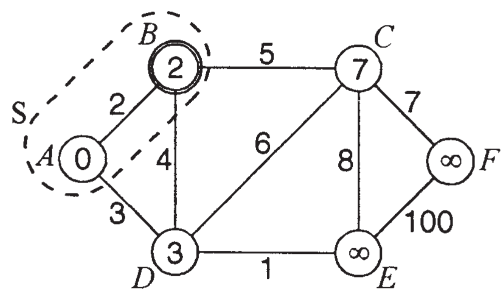

Figure 4. Node B is added to S and its neighbors are relaxed.

For the next iteration, the node outside of S with the lowest d value is chosen. In this case, we consider D since $d(D) < d(C)$, $d(D) < d(E)$, and $d(D) < d(F)$. By choosing D, the neighbors of D are relaxed. Node C could be reached in $3 + 6 = 9$ units of time by going through D, but $d(C)$ had already the value 7, which is less than 9, so $d(C)$ remains unchanged. On the other hand, $d(E)$ is changed to 4 since the value of d for E was infinity (see Figure 5). Now D is added to the list S since all of its neighboring nodes have been relaxed.

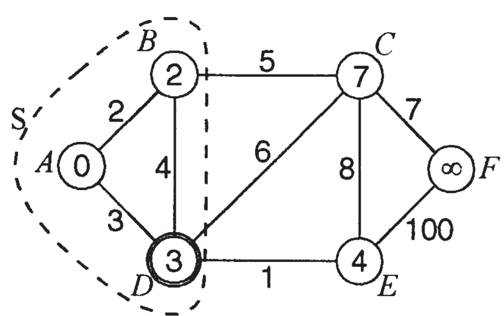

Figure 5. Node D is added to S and its neighbors are relaxed.

For the next iteration, we consider E since $d(E)$ has the smallest d value among the remaining nodes. Node C can be reached by E in $4+8=12$ units, but $d(C)$ is unchanged as it had already been assigned a lesser value of 7. During this iteration, $d(F)$ will change to $4+100=104$, the length of the path that runs from A through E to F, since before $d(F)=\infty$. Node E is now added to S (see Figure 6).

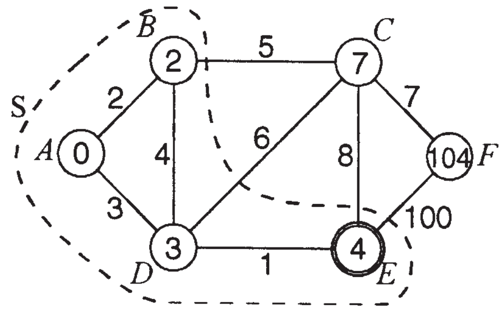

Figure 6. Node E is added to S and its neighbors relaxed.

Although there is now a known path to the destination node F, there are still some nodes left outside of S, namely C and F. Choosing C from the list, the neighbors of C are relaxed. The current distance assigned to $F, d(F)=104$ is greater than the length of the path that runs through C, which is $7+7=14$. Therefore, $d(F)$ is updated to 14 (see Figure 7). Node C is then added to S.

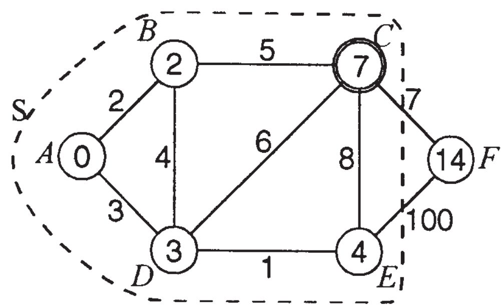

Figure 7. Node C is added to S and its neighbors relaxed.

Finally, the only node left not in S is F, but the relaxation of the neighbors of F is trivial (see Figure 8). Dijkstra's Algorithm would now report that node F can be reached from node A in 14 units of time. As mentioned previously, Dijkstra's Algorithm does not stop simply when a path from the source to the destination is found, but only when the shortest paths from the source to every node on the graph have been found.

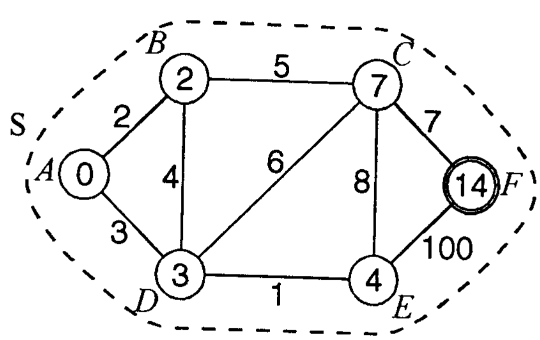

Figure 8. The final trivial relaxation of node F

Dijkstra's Algorithm calculates shortest distances to each node in the graph, but it can also keep track of the actual shortest path from the source to the destination node in the following way: A "mentor slot" is created for each node that records the node preceding it (from which its value of d was most recently updated). In the example above, B's mentor is A and F's mentor is at first E but then is changed to C. Once Dijkstra's algorithm has finished, the path is traced backwards, starting at the destination node.

In truth, not every node must be examined in the case that all weights are nonnegative numbers, as is the case with road networks. If, during the computation, a path is found to the destination location with a certain distance and all remaining unchecked nodes have known distances from the source that are greater than that distance, then there is no reason to continue looking. Intuitively, if the source and destination locations are far apart in the road network, then a large number of nodes must still be examined and relaxed. In the worst scenario the destination node will be the last node to be examined, causing every node to be checked.

Chapter 4
Level Graph Algorithms

The Level Graph Search and Augmented Level Graph Search algorithms were chosen because they can find approximate solutions to the shortest path problem quickly. These algorithms require information on the road classification, which is readily available in most digital maps.

4.1 Motivation

In the process of developing algorithms to solve the shortest path problem, a natural step is to observe how humans look for a route on a map from one place to another. Consider a trip from Harvey Mudd College in Los Angeles to the Statue of Liberty in New York City. Using Dijkstra's Algorithm, the majority of the nodes and edges that are checked have very little chance of contributing to the optimal path. Any rational person would not consider paths through residential streets in Seattle, Washington since they would not be good paths to use to get from Los Angeles to New York.

Intuitively, on any good route for a long trip most of the time should be spent on the superhighways, and when not on superhighways more of the remaining time should be spent on state roads rather than dirt roads. Dijkstra's Algorithm is rather simple-minded in this respect since it treats every road in the same manner, regardless of whether a road is a superhighway or a dirt road. Fortunately, almost all digital road networks contain road classification information, which could be used in the following manner: First, find a path to an interstate from the starting point in Los Angeles. Next find a path to an interstate from the destination point in New York. Finally, find a path between the two that uses only the interstate system. Dijkstra's Algorithm could be used for this final search stage, since the number of interstate roads is small enough that there are no performance problems due to memory constraints.

4.2 LGS: Level Graph Search

In 1992, these ideas were formalized by Shapiro, Waxman and Nir into the Level Graph Search (LGS) algorithm ("Level graphs and approximate shortest path algorithms," *Networks*, 22:691–717, 1992). LGS operates on the principle of spending as little time as possible on roads that are not superhighways. Consider the example above of traveling from Harvey Mudd College to the Statue of Liberty. The starting location would be on 12th Street near Dartmouth in Claremont. The first step in LGS is an algorithm called Level Path Search (LPS), which operates somewhat like Dijkstra's algorithm until a better road such as Foothill Boulevard is encountered (see Figure 9).

Once that road is found, LGS ignores any roads that are smaller than Foothill Boulevard. Also, LGS records the path from the starting location to where a better road was encountered. The same process is now repeated: roads of the same class as Foothill Boulevard are searched using LPS (see Figure 10).

Now that an even better class road is found (in this case, Interstate 10), the path to it is marked and a new search is performed only on the roads of the same better class. This process terminates when a superhighway has been reached from the starting location. Next, this same procedure is applied to the destination point in New York. Finally, a connecting path is found between the starting point (Harvey Mudd College) and the destination point (New York City's Statue of Liberty) using Dijkstra's Algorithm on the network of superhighways (see Figure 11). Dijkstra's Algorithm is used for this final step because

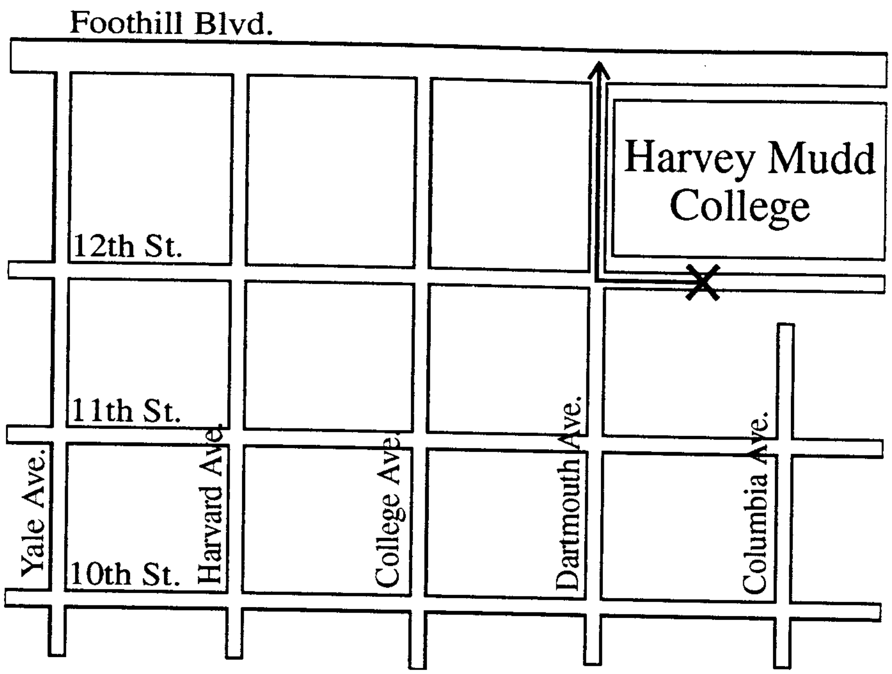

Figure 9. The search for a better class of road starting from 12th and Dartmouth

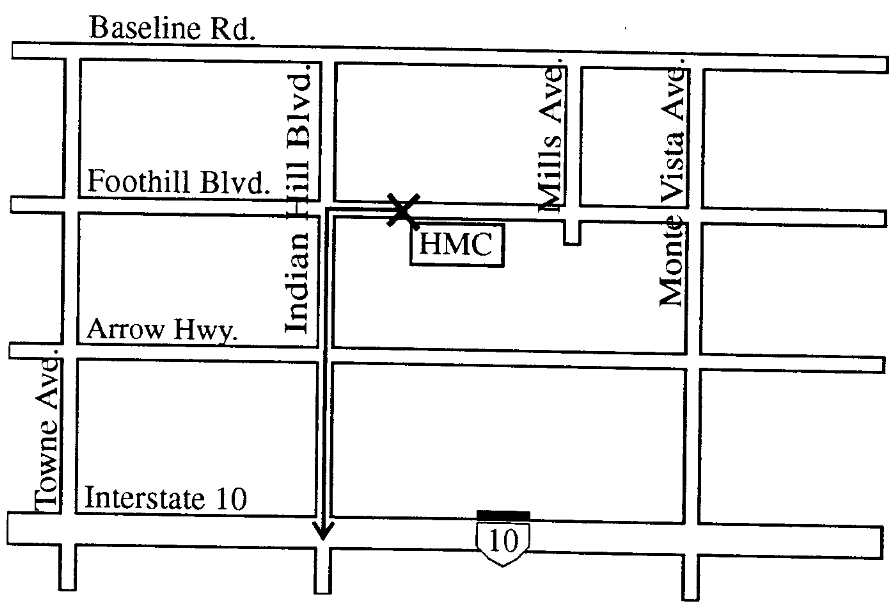

Figure 10. The search for a better class of road starting from Dartmouth and Foothill

the number of superhighways is a relatively small number of roads when compared with the total number of roads in the entire United States. As a large number of roads have been ignored, it is this final step where most of the computational savings are obtained.

4.3 ALGS: Augmented Level Graph Search

Under certain circumstances, LGS suffers from two problems which cause it to generate poor paths. This first problem, known as the near-path problem (see Figure 12), is due to the fact that LGS always seeks

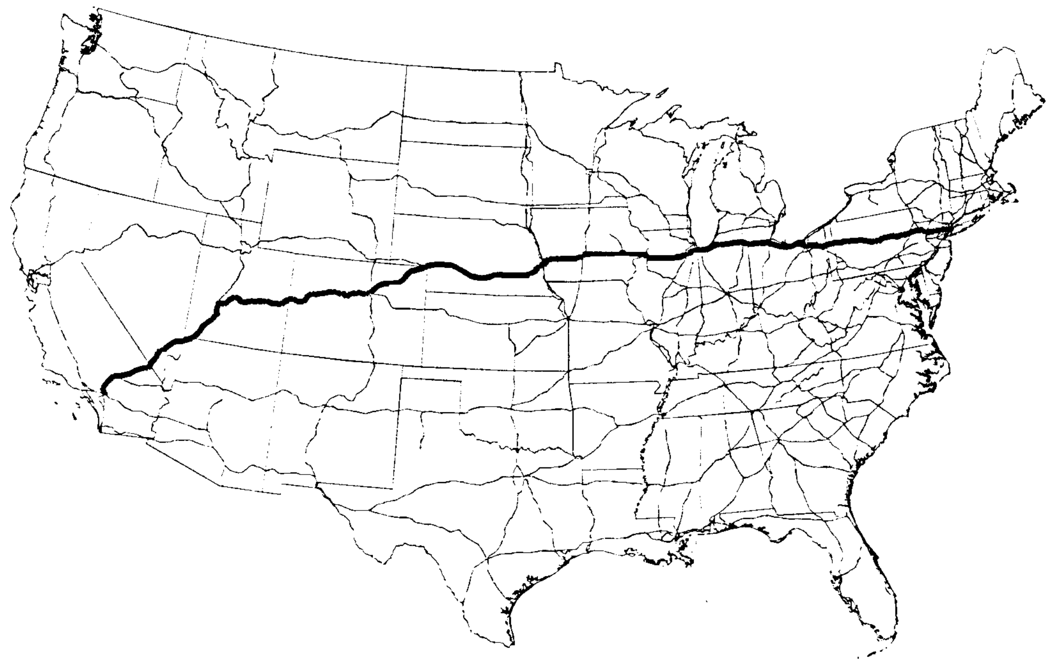

Figure 11. The search for a connecting path along interstates between Harvey Mudd and the Statue of Liberty

out a superhighway regardless of the actual distance between source and destination locations. Clearly it seems unreasonable to travel on a superhighway if one only desires to get to the grocery store a few blocks away.

The second problem with LGS is that it assumes that there is always a way to get to the next better class of road using only the roads of the class under consideration. In other words, it is never necessary to degrade the current class of road in order to get to a better class of road. For a road network, an example of such a situation might involve a superhighway whose on-ramp is only accessible by first traversing a side street.

In response to these shortcomings, Shapiro, Waxman, and Chen created the Augmented Level Graph Search Algorithm, or ALGS ("Near-point heuristics for rank constrained level graphs," *Congressus Numerantium*, 89:15–32, 1992). The main difference between LGS and ALGS is that ALGS relaxes the constraint of always looking for the closest better-class road. Unlike LGS, the goal of ALGS is not to find a path on the interstates, but on any class of road that is suitable. LGS finds a path to a highway from the source node, and then from the destination node, and then connects the two points using the highways. ALGS applies the LPS algorithm to both source and destination nodes simultaneously, and always confining both searches to the same class of roads. That is, once the next better class of road is found from the source node, the search is suspended until the same class of road is reached from the destination node. When both searches are at the same class, ALGS applies a heuristic function to check the likelihood of the existence of a connecting path between both nodes using the current class of roads. In order to determine if a connecting path using the current class of roads is desirable without actually computing the connecting path, the heuristic function estimates the impedance of the connecting path.

As an illustration, consider the network shown in Figure 12. The source and destination nodes are S and D, respectively. The LPS algorithm is used to find the next better class of road, which occurs at nodes A and B. At this stage, the heuristic is used to decide whether to continue searching for better class roads, or keep searching on roads of the current class used to get from S to A and D to B. In particular, there are two heuristic functions: $h_b(m, n)$ for the approximate distance between nodes n and m along roads of the better class (hence the subscript b), and $h_c(m, n)$ for the approximate distance between nodes n and

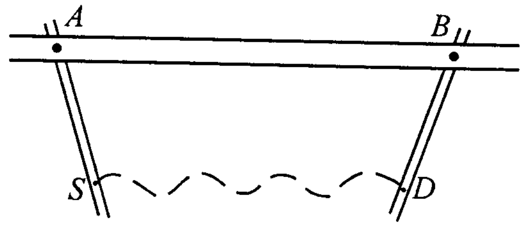

Figure 12. A typical near-path problem

m along roads of the current class (hence the subscript c). The actual distances $d(S, A)$ between S and A, and $d(D, B)$ between D and B are already known since they were obtained when nodes A and B were found. To decide if there is a good chance of finding a path that stays on the current class of roads, ALGS checks whether the following inequality is true:

$$h_c(S, D) < d(S, A) + h_b(A, B) + d(D, B).$$

If so, then a search is performed on the roads of the current level for a path connecting node S and D. If false, ALGS continues searching for the next better class of roads (as LGS would have done).

The choice of the heuristic function is crucial. If a particular heuristic h tends to overestimate travel time, ALGS would behave like LGS, because even for close points it would be unlikely to check for local paths. If, on the other hand, h tended to underestimate travel time, ALGS would operate similarly to Dijkstra's Algorithm because it would almost always search for local paths, thereby defeating the purpose of trying to save memory by excluding from consideration as many roads as is reasonably possible. Thus, selecting a suitable heuristic measure is important to the successful operation of the ALGS algorithm.

Furthermore, the heuristic function is dependent on the configuration of the network and intended application of the ALGS algorithm. Fundamentally, the heuristic is level dependent so there are different heuristic functions for each level. For example, the estimated travel time along surface streets might be based on an average speed of 35 miles per hour, whereas the estimated travel time along interstates is based on the 65 miles per hour speed limit. In addition, if the overall goal is to reduce the estimated travel time for a delivery vehicle, it might be important to take into account the time for truck stops and weigh stations when travelling along interstates. Naturally, these factors don't affect the estimated travel time along surface streets so the two heuristic functions for interstates and surface streets are very different.

Chapter 5
Discussion of Results

The procurement of test coverages and the development of a visual interface between the finished code and ARC/INFO was necessary before comparisons could be made between Dijkstra's Algorithm and the Level Graph Algorithms. Heuristics were not implemented, so there were no tests involving ALGS. It was observed that LGS computed short paths from 2 to 6 times faster than Dijkstra's Algorithm for large networks.

5.1 Test coverages

To compare the efficiency and accuracy of Dijkstra's Algorithm and the Level Graph Algorithms, the clinic team obtained networks that would best represent the profiles of ESRI's customers. Networks with 100,000 to 500,000 nodes were desired as this is the typical size of a network in industrial uses. ESRI supplied the clinic team with digital maps of Redlands, California and a larger map of Southern California, with about 10,000 and 160,000 nodes respectively. The team devised several other simulated networks that ranged from 800 to 500,000 nodes, all based on large, regular grids of roads (see Figure 13). The simulated networks were convenient because they could be made as large as desired, but they do not represent road networks very well.

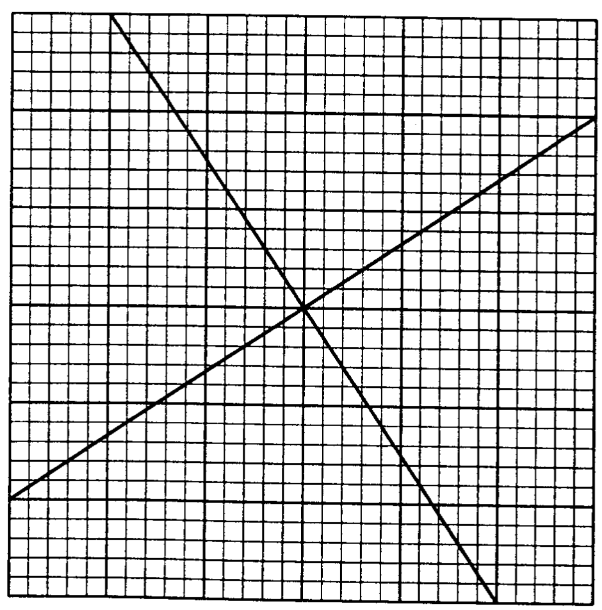

Figure 13. A simulated road network based on a regular grid

5.2 Demonstration interface

A large portion of the clinic's time was devoted to the development of an interface between ARC/INFO and the external solver code written in C. As this current implementation of ALGS does not directly access the information stored by ARC/INFO, all information is passed between ARC/INFO and the code via ASCII text files (see Chapter 6). The interface, written in the native macro language of ARC/INFO, automates the manipulation of these text files. The interface also allows for the visual selection of source, destination and intermediate stops by clicks of the mouse on the display of the map as well as the display of the calculated solution and computation of statistics on efficiency and accuracy. Furthermore, the interface ensures that the same source, destination and intermediate stops are selected for comparison tests.

A listing of all files comprising the visual interface is given in Appendix A. The interface is invoked with the command ``&run path'' which must be issued in the ArcPlot environment (a subset of ARC/INFO). A pop-up menu will appear that will allow the user to choose the desired coverage (road network). After the coverage is displayed, the user has the option to select the source, destination, and intermediate stops either by clicking on the map or by entering coordinates using the keyboard (see Figure 14). The stops may also be retrieved from a file. The points chosen do not have to be nodes and may lie on an arc. In addition, it is possible to specify more than one route to be solved.

When all of the routes have been entered into the computer, the coordinates of the stops for the different routes are exported into an ASCII file. The user then has a choice of which algorithm to use to solve the problems. After the selected algorithm determines the shortest path between the prescribed points, the information is imported back into ARC/INFO in the form of route attribute tables (RAT) and section tables (SEC). The calculated paths are displayed along with timing information, statistics on routes, and directions on how to traverse the routes.

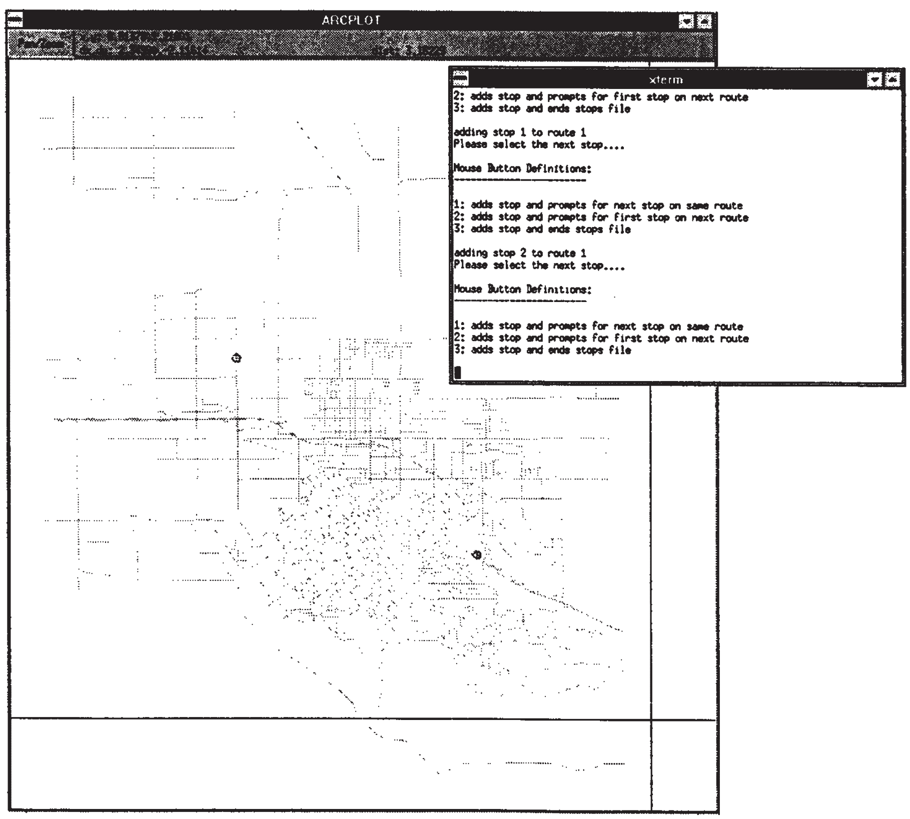

Figure 14. Sample screen shot of the visual interface showing the road map of Redlands

5.3 Heuristics

Recall that the heuristic is an estimation of the impedance between two points for a given class of road. As noted in Chapter 4, the heuristic is dependent on the type of network chosen and the intended application of the shortest path problem. Consequently, it becomes impossible to devise a general heuristic function.

The clinic team chose to begin its comparisons between ALGS and Dijkstra's Algorithm using a heuristic based on the air distance between two points in a road network. This heuristic estimates the travel time by dividing the air distance between two points by the speed limit of a particular road class. One difficulty encountered in devising this particular heuristic is that all of the roads in a particular road class do not necessarily share the same speed limit. Secondly, for the map of Redlands and of Southern California, the units of length and impendance were difficult to ascertain. Third, the calculation of air distance requires that the location of nodes be given in x- and y- coordinates, which were not known for the map of Southern California. Faced with these unresolved issues, the clinic team was not able to fully implement the air-distance heuristic for ALGS in time.

5.4 Test results

Currently, the only information about the speed of the algorithms is the amount of time that each process of the solver takes to complete. These numbers can be inaccurate because the computer's performance will vary depending on the system load at the time of the test. CPU time, the actual amount of time spent by the computer executing the solver code, is preferable. In addition, statistics on the amount of memory swapped would be useful. The clinic team will work with ESRI during May 9–10 to implement these timing routines.

Because in the current implementation the Level Graph Algorithms do not interface directly with ARC/INFO, it was necessary to transfer information in the form of ASCII files. The solver code actually performs four functions: 1) it builds a network directory (internal data representation used by the Network Engine) if necessary, 2) reads in the stops file (see Section 6.4), 3) solves for the shortest path, and 4) outputs the path. To compare the efficiency of the algorithms themselves, only the time taken for the actual computation of the path will be used. The time required to read in the stops file and output the path is very small in comparison to the computation time. Moreover, the time for the construction of the network directory will not be considered since this step is only performed once for any network of roads (see Section 6.3.3).

Tests were performed on a HP 9000 series computer (the HP 715) running at 100 Megahertz. The computer had 32 megabytes of memory and 128 additional megabytes in swap space. The solver code was compiled with optimization level 2 and debugging output was turned off (see Section 6.1.2).

As the heuristics have not been implemented, statistics for ALGS are not included. For each of the six tests shown in Table 1, the source and destination nodes were chosen relatively far apart in the road network. This was to avoid the near-path problem discussed in Section 4.3. To reduce the errors associated with fluctuating system load, multiple trials were performed and the resulting times averaged.

The test results show that for small road networks, there are no substantial savings in time when LGS is used. In fact, for Grid #1 (a 20×20 simulated network), and the map of Redlands (1,814 nodes) there are no detectable savings in time and the calculated paths are sub-optimal. Therefore, it seems that Dijkstra's Algorithm performs better overall for these two road networks. For the map of Southern California with approximately 140,000 nodes, LGS performs four to five times faster than Dijkstra's Algorithm and produces paths that are 3% to 10% longer than optimal. The final test using a simulated network with about 230,000 nodes shows LGS performing about twice as fast while generating paths that are very close to optimal.

The clinic team expects that ALGS will generate paths that are closer to optimal, while still operating substantially faster than Dijkstra's Algorithm.

Test No.	Coverage Name	Dijkstra's Algorithm		LGS Algorithm		Comparison	
		Time (secs)	Total Impedance	Time (secs)	Total Impedance	% Over Optimal	% Speed Increase
1	Grid #1	< 1	143.5	< 1	149.5	4.2	n/a
2	Redlands, CA	< 1	1762.7	< 1	1872.9	6.2	n/a
3	Redlands, CA	< 1	1243.3	< 1	1603.7	28.9	n/a
4	Southern Calif.	18	66.6	4	73.6	10.5	77.8
5	Southern Calif.	28	117.9	3	121.6	3.1	89.2
6	Grid #2	148	1470.1	79	1479.1	< 1	46.7

Table 1. Test results comparing time and efficiency of Dijkstra's Algorithm to LGS and ALGS

Chapter 6
Solver Code

This chapter describes the set up and use of the solver code developed by the clinic team, as well as guidelines for selecting and maintaining coverages for use with the Level Graph Algorithms.

6.1 About the finished code

The Level Graph Algorithms have been implemented in C as external solvers for the Network Engine (see Section 1.3). Currently, the code interfaces with ARC/INFO through the passing of ASCII (text) files, but in the future this step will be unnecessary since it will become possible to access information in ARC/INFO directly using the Network Engine.

The finished code that will be delivered to ESRI falls into three categories. The first is the set of programs used to read the ASCII files that describe the network, translate them into an internal representation (the one provided by the Network Engine), call the solver routine, and output the solution. There are two of these files, one for LGS (called `lnetload.c`) and one for ALGS (called `anetload.c`). The two algorithms need different loaders because ALGS needs one more text file (one describing where the nodes are located physically) than LGS does. The two netload files are derived from a pre-existing file, `netload.c`.

The second category of file is the solver file. There was a pre-existing solver (one that used Dijkstra's Algorithm) called `path.c`, which was the model for `lgs.c` and `algs.c`. These files use the same interface as `path.c`. Because ALGS depends on a heuristic that needs to be adapted to the individual problem, the heuristic code was put into a separate file named `heuristic.c`. Currently, changing the heuristic involves modifying the file `heuristic.c`, re-compiling, and re-linking all the object files.

The third group of files contain changes made to the Network Engine (see the following section). These include `netlib.h, netio.c` and `netview.c`.

6.1.1 Technical details on layers

In implementing the Level Graph Algorithms, several new procedures were added to the Network Engine to allow for the manipulation of data structures known as layers. Layers are simply groups of arcs and nodes in which levels are in a one-to-many relationship with arcs and nodes (i.e., each layer contains many arcs and nodes but not vice versa). Although layers can be used for a variety of purposes, the clinic team has used them to implement the level hierarchy of arcs in a network. Levels are the user-defined ordering of edges in a network (see Section 6.3.1). Levels and layers correspond naturally, except for the fact that in Network Engine all arcs are unidirectional whereas in ARC/INFO all arcs are bidirectional. If all arcs of the same level were lumped together, the original orientation of all arcs would be destroyed. (Even though an arc is bidirectional, it still has a "from" node and a "to" node which gives it a specific orientation.) Therefore, each bidirectional arc will be added twice, forming two separate unidirectional arcs in separate layers to keep track of which direction of the arc is the original direction.

The standard procedure devised by the clinic team, for a given bidirectional arc with level L, is to store its unidirectional arc pointing in the original level L into layer number $2 \times L$, and to store the arc in the opposite direction into layer $2 \times L + 1$. This means that the lowest layer number that is used is 2, since the smallest level is 1. To convert from the layer number of an arc to its original level, simply use floor(LayerID/2).

In the Network Engine, the most fundamental operation on arcs is the ForwardStar query which provides a list of all arcs that are anchored to a particular node. The Level Graph Algorithm requires a more complicated query that returns arcs that are in a certain range of levels. (This is how the Level Graph Algorithms ignore lower classes of roads.) The original ForwardStar query has been modified to allow for this feature using a lookup table for the LayerKey, which is the unique identification number that is assigned to the layer by Network Engine. Essentially, if the layer with LayerKey L is eligible to be returned from a ForwardStar query, then array location L in the lookup table is non-zero. The lookup table is not a bitmap; a full 32-bit integer is used for each location. This wastes a little bit of memory, but since there should never be any more than 100 layers this scheme uses less than 1 kilobyte of memory.

To manipulate the lookup table, four new functions have been added to the Network Engine:

```
NE_AddAllNetviewLayers()
NE_DelAllNetviewLayers()
NE_AddNetviewLayer()
NE_DelNetviewLayer()
```

For further information on these functions, refer to the header file `netlib.h`.

6.1.2 Compiling and Linking

There are some C preprocessor variables that can be defined at compile-time to affect how the solver code compiles. The most prominent of them is `NODEBUG`. If this is defined, the executable will not generate any output for debugging purposes. This speeds up the execution a little bit, and looks better. Another variable is `OPT_ LEVEL` which doesn't affect how the program works but indicates the optimization level at which the program was compiled. If compiling is done with -O3, for example, then `OPT_ LEVEL` should be defined as "O3" (including the quotes). This helps to ensure that the time comparisons between the solvers are accurate since it wouldn't be fair to compare something compiled with -O0 (no optimization) to something compiled -O3. To find the optimization level, use the UNIX command "`strings`" on the executable and look for the words "Optimization Level."

The basic idea behind the linking of the solver code is to link `lnetload.o` with `lgs.o`, and `anetload.o` with `algs.o` and `heuristic.o`. Makefiles have been created to control the compiling and linking of the programs. Wildcards like "`*.o`" cannot be used in the Makefile because they will cause `path.o` to link with `anetload.o`.

To install the Level Graph Algorithm solvers on a source tree,

1. Integrate changes in `netlib.h, netio.c, fstar.c`, and `netview.c` into the current source tree,
2. Copy the new files into `solvers` and `netload` subdirectories,
3. Compile all of the modified or new files and link them using the Makefile.

The new netload programs need two extra header files: `<time.h>` and `base/include/memsys.h`. For a more complete file listing, see Appendix A.

6.2 A word on coverages

In ARC/INFO terminology, the network of roads on which the calculations are to be performed is called a "coverage." As the results obtained from any analysis performed on a coverage are only as accurate as the coverage itself, it is important that the coverage is carefully chosen and maintained. For the Level Graph Algorithms, coverages should accurately represent roads and all of the connections between the roads. In the case of road networks, this is especially important because some coverages are not meant to represent connections between roads (i.e., the topology) but rather give the visual layout of the roads. For example, the TIGER digital maps published by the United States Geological Survey are not topologically accurate because they represent any road that passes under a bridge as two disconnected roads, thereby conveying the correct aerial viewpoint but misrepresenting the actual connectivity of the roads.

Along these lines, it is desirable for all interstates and highways to be undivided; i.e., the two directions of travel must not be represented as separate edges. The reason for this condition is that the highway system becomes a disconnected collection of one-way roads. For example, Figure 15 depicts a shortest path problem in which the source and destination locations are on opposite sides of a divided highway. The arrows in the figure indicate the direction of traffic. When LGS or ALGS searches for the closest interstate from the source location, it will begin travelling along the westbound side of the highway, although it really should be travelling to the east. If the highway was not divided (i.e., represented as a single, bi-directional road), it would be possible for either algorithm to correct itself by turning around. (The wrong on-ramp would have been used, but this does not adversely affect the calculated path.) However if the highway was divided there would be no way to "switch sides" without traversing on a lower class of roads. In this case, LGS would simply fail; ALGS would try to use surface streets, resulting in a poor solution.

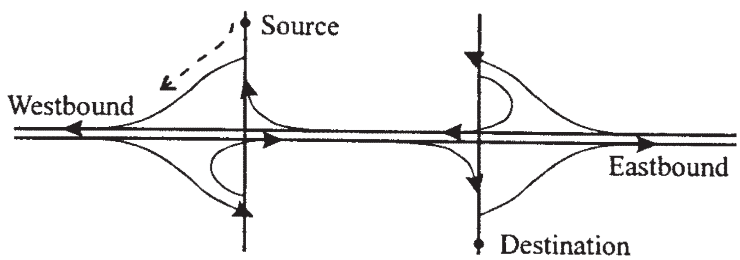

Figure 15. One possible problem caused by a divided interstate

If a coverage has interstates that are divided, there is little that can be done without much effort. One possible solution is to delete one side of the freeway and make the remaining side bidirectional, along with all adjoining onramps and offramps. The single remaining freeway will not represent the actual freeway, but it will allow for more meaningful results from the Level Graph algorithms. A less cumbersome solution is to add zero-impedance u-turns between separated highways near the onramps so that if LGS or ALGS chooses to travel in the wrong direction, it can correct itself without greatly affecting the total impedance.

6.3 Preparation of coverages

Before using the solver code, there are a number of preliminary steps to perform. First, the arc and node attribute tables (ARC/INFO's internal representation of the arc and node topology) must be present in the coverage. Second, all of the necessary items (fields) must be prepared. Each arc should have two impedances representing the cost of traversing that arc in each direction and a level code (see Section 6.3.1). It should be remembered that the impedance represents the value that is to be minimized, and that a negative impedance for an arc means that travel is not allowed in a particular direction along that arc. Third, to enable the selection of stops at any point along an arc, a route system for the entire coverage must be created in which each arc has its own route. The final step is to export all of the arc and node information into text files (see Section 6.3.3).

The AML script `covformat.aml` automates the creation of the route system and the exporting of the arc and node topology. When executed with the "`&run covformat`" command, the macro will present the user with a dialog box to select the coverage desired. If the route system or topology files already exist, the macro will not overwrite them.

6.3.1 Preparing level codes

As the main idea behind the level graph algorithms is the computational savings gained by the use of road class information, it is important that this information is correctly conveyed to the Level Graph Algorithm solvers. The "level code" is a positive integer that represents a particular class of road. The solver expects that a level code of 1 represents the interstates, the best road class. Other classes of roads should be numbered sequentially, the highest number representing the worst road class. In general, the roads represented by each level code should have roughly the same speed limit or average travel speed.

6.3.2 Third-party digital maps

In most cases, the data that are used come from a third-party source and has its own road classification scheme. It is then necessary to devise a mapping that converts the third-party road classifications into suitable level codes for the Level Graph algorithms. The simplest way to achieve this is to create a look-up table for this purpose.

To illustrate how this mapping is created, the procedure used by the clinic team to prepare the test data will be described. The test data are based on the popular TIGER format, developed by the United States Geological Survey. In total, there are 74 different road classifications in the TIGER format specification, although in the data only 23 of these are actually used. Table 2 shows the breakdown of these road classifications and their mapping into level codes. It should be noted that the collection of all level 1 roads represents less than 5% of the total number of roads, so that the Level Graph algorithms are very effective.

Level Code	Road Class.	Frequency	Description
1	A10	1,075	interstate
	A11	350	interstate, unseparated
	A15	48	interstate, separated
	A20	665	state highway
	A21	1,819	state highway, unseparated
	A23	1	state highway, unseparated, underpass
	A25	1	state highway, separated
	A28	27	state highway, rail line in center
	A63	1,955	access ramp
2	A30	10,301	secondary road
	A31	5,482	secondary road, unseparated
	A33	2	secondary road, unseparated, underpass
3	A40	80,199	local road
	A41	43,847	local road, unseparated
	A42	3	local road, unseparated, tunnel
	A43	4	local road, unseparated, underpass
	A45	142	local road, separated
	A70	29,550	other thoroughfare
	A60	26	unspecified
	A62	6	roundabout
4	A50	104	vehicular trail
	A51	239	vehicular trail, unseparated
5	A71	12	walkway for pedestrians

Table 2. Breakdown of road classifications for the sample TIGER data and mapping into level codes

Another common problem of digital road information is the inevitable inaccuracy or incompleteness of data. For example, in the sample data there are almost 30,000 roads of unknown class (A70). Undoubtedly, this is due to lack of information to place these roads into their correct categories. The problem then is to assign these roads a level code without any information on road classification. In this case, the clinic team chose to include these roads into level 3, which forms the majority of all roads. In this way, the number of incorrectly assigned roads is likely to be minimized. Furthermore, a visual inspection of this classification of roads showed that these roads were randomly scattered and were not interstates.

6.3.3 Exporting the coverage topology files

As the current implementation of the Level Graph Algorithms does not access the coverage directly, it is necessary to first export the coverage information into text files so that the code can read them and build a network directory: Network Engine's internal representation of the coverage information. The network directory is not erased after the completion of the solver, so that if no changes have been made to the coverage, the code will begin finding the prescribed short paths without first building the network directory.

The necessary fields from the arc and node attribute tables of the coverage should be exported as space-delimited ASCII files in the formats given in Tables 3 and 4.

From-Node	To-Node	From-To Impedance	To-From Impedance	Level	Edge ID
FNODE#	TNODE#	FT-IMPED	TF-IMPED	LEVEL	UID

Table 3. Format for the exported arc attribute topology file

Coverage Node ID	x-coordinate	y-coordinate
< *coverage* >#	X-COORD	Y-COORD

Table 4. Format for the exported node attribute topology file

In the arc attribute table, the UID field is a user-specified edge identification number (which must be a unique integer) that is used when creating the section tables after the calculation of the path. The exporting of coverage topology files may be accomplished by the `unload` command in the Tables environment.

6.4 Stops file

In addition to the exported node and arc attribute information, a "stops file" defining the shortest path problem must be passed to the external solver. The format of the stops file is given in Table 5. The source, destination and intermediate stops do not have to lie at a node and may be situated anywhere along an arc. In order to specify the location of a stop along an arc, the ALGS solver requires the from- and to- nodes of the arc and the percent along the arc where the stop lies. A value of zero for the percent indicates that the stop actually lies at the from-node; a value of 100 places the stop at the to-node.

From-node	To-node	Percent along arc	Order	Route identifier

Table 5. Format of the stops file

The clinic team's implementation of the Level Graph algorithm allows the user to specify more than one route to be solved at a time. Each route is assigned a unique integer identifier and must have at least two stops on it. Every stop must bear a route number as well as the order number which signifies the position of the stop along the route. Intuitively, an order number of 1 signifies that the stop is the source. However, the order numbers do not have to be consecutive and do not have to start at 1. The stop with the lowest order number for a given route is the source and the stop with the highest order is the destination.

6.5 Execution of the solver code

To execute correctly, the solver code must be provided with the arc and node topology files, the stops file, the name of the coverage, and the desired filenames of the output section table and a route table (both in ASCII) that contains the solution(s). This configuration is depicted in Figure 16.

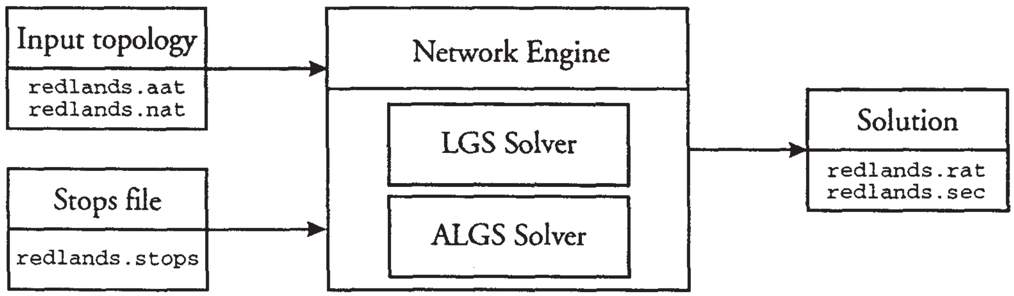

Figure 16. Schematic of solver code

The ALGS solver code must be called from the command line as follows:

`algspath.exe` *<export arc attributes filename><export node attribute filename>*
<stops list filename><coverage name><output route table filename>
<output section table filename>

The LGS solver code must be called from the command line as follows:

`lgspath.exe` *<export arc attribute filename><stops list filename>*
<coverage name><output route table filename>
<output section table filename>

Note that the LGS solver code does not need the node attribute topology file.

Upon completion, the solver code will place the information for calculated paths in a section and route table (both ASCII). The output route table will simply be a list of all the route identifiers that were used in the stops file. The section table will be in the standard ARC/INFO format (see the ARC/INFO manual for more information).

Chapter 7
Conclusion

7.1 Recommendations for future work

The next step in the implementation of the Level Graph Algorithms is to code a heuristic function and to test the accuracy and speed of ALGS with LGS and Dijkstra's Algorithm. Once the implementation is completed, the algorithms should be better interfaced with ARC/INFO through the Network Engine to eliminate the passing of text files.

As noted in Chapter 4, the heuristic used by ALGS is dependent on many factors which depend on the coverage used. Currently, the heuristic is "hard-coded" into the file `heuristic.c` so that if any changes are made to the heuristic, all executable files must be compiled and linked again. Since the user of the solver will not want to go through this procedure each time a different coverage is used, a more intelligent heuristic program should be devised so that the user can change the details of the heuristic without recompiling. One possible solution might be to encode the heuristic measure in some general format and pass it to the ALGS solver just as the attribute and node topology files are passed through using ASCII files.

A useful addition to the solver code would be the implementation of turn costs, the impedances associated with the interchanges between arcs. Essentially, if there are n arcs anchored to a node, there are n^2 turn costs for that node representing all possible combinations of arc combinations. If a turn is not allowed from one edge to another, then the associated turn cost should be a negative number. For road networks, turn costs are actually quite significant in the determination of the path with least impedance. The implementation of turn costs is a relatively simple task. Far more difficult is actually obtaining turn cost data for large sets of roads.

Although the Level Graph algorithms have been tested on road networks (by far the most common networks), they have not been tested on other types of networks, like networks of rivers or electrical circuits. Tests have been performed on simulated networks, which probably most resemble electrical circuits. It would be interesting to see if there are any differences in the execution of the code for different types of networks.

It might also be of interest to investigate whether there is an optimum number of levels for the Level Graph algorithms. Intuitively, it seems that if there are only one or two levels, then the Level Graph algorithms will not be fully exploited. However, if there are too many levels, then the algorithms may become too inaccurate.

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Appendix A
File Listing

The following is a listing of all files that make up the visual interface. The main program, `path.aml`, should be called from ARCPLOT. All other files are used by `path.aml` and should not be executed independently.

```
path.aml
aml/findpaths.aml
aml/route_import.aml
aml/getstops.aml
aml/routemplate.aml
aml/clearoutes.aml
aml/initmap.aml
aml/directions.aml
amlpathalg.aml
aml/stat.aml
aml/displaypath.aml
aml/stopstables.aml
```

The AML script `covformat.aml` helps to prepare a coverage for use with the solver code. It has no other associated files and should be executed from ARC/INFO.

The solver code consists of the following files:

```
solvers/lgs.c
solvers/algs.c
solvers/heuristic.c
netload/lnetload.c
netload/anetload.c
```

These following files contain changes made to the Network Engine for the implementation of the Level Graph Algorithms:

```
include/netlib.h
netio/netio.c
nettrav/fstar.c
netview/netview.c
```

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