Social and Behavioral Sciences

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Introduction

Mathematics for the Social and Behavioral Sciences is a very broad field of study, including applications in the social sciences (anthropology, demography, economics, education, geography, history, linguistics, political science, sociology) and in the behavioral sciences (psychology and cognitive sciences). This area is tightly allied with other program areas in the CUPM, including Financial Mathematics, Applied Mathematics, Statistics, as well as the areas of Mathematical Economics and Mathematical Modeling. Some of the same types of analysis, on similar or related problems, are used for understanding social and behavioral sciences.

The exciting aspect of this area is that despite the hundreds of years of development of mathematical tools to study the sciences, and, for a shorter period of time, mathematical economics, many of the mathematical tools, models, approaches, and frameworks to study social and behavioral sciences have only been studied for the past thirty years and many have yet to be developed. Mathematical analysis has been used to study demography, voting theory, the fair division of goods, game theory, social networks, analysis of conflict, social choice, measurement and learning (from mathematical psychology), and other topics. This area can expect to see tremendous growth as professionals from a number of fields realize the value of mathematical analysis.

History

The use of mathematics for social and economic analysis dates back to the 17th century when the presentation of government and economic data gave rise to descriptive statistics. Mathematical economics gained solid footing in the 19th century. Irving Fisher, an American economist active in the late 19th century, finished his 1929 Josiah Gibbs lecture by saying "Mathematical notation is, as Gibbs said, simply a *language*. It is required for the best expression of scientific method when the relations to be expressed become sufficiently involved to require it in preference to ordinary language, which is less precise and complete. The outlook is bright for a healthy development of mathematics in the social sciences."

Fisher's vision of the usefulness of mathematics to the social and behavioral sciences was certainly correct, and the mid- to late-twentieth century saw fruitful mathematical excursions into those fields to explain some known observations. Moreover, the more mathematics was used, the clearer it became that mathematics was a powerful tool in both

understanding and discovering a number of phenomena previously unexplained without mathematical analysis.

What should undergraduates see?

Undergraduates interested in applying mathematics to help solve the world's problems—from analyzing the fair division of lands or resources, to studying the demography of a people over time and space, to constructing a government that fairly represents the will of the people, to quantifying the interconnectedness of people as a rumor or disease spreads through a population—will find interest and enthusiasm for the mathematics which allows them to investigate and analyze these important issues.

Mathematics courses that teach undergraduates to analyze and understand data, patterns, and models of the world around them are appropriate for any major in social or behavioral sciences with a particular interest in mathematics or a general interest in an advanced degree in their field, as well as for any mathematics major with an interest in applications to the social or behavioral sciences.

To demonstrate the applicability of mathematics and career options, all mathematics majors should be exposed to applications of mathematics to the social and behavioral sciences in calculus, linear algebra, discrete mathematics, modeling, and graph theory courses.

Undergraduate preparation for students interested in focusing their mathematics major in the social or behavioral sciences

Students interested in a mathematics major that emphasizes social or behavioral sciences should take coursework that includes the following elements:

1. Data Analysis and Visualization
2. Probability and Statistics
3. Mathematical Modeling
4. Discrete Mathematics (Combinatorics, Ordering Relations, Graph Theory)
5. Game Theory, with a focus on applications
6. Computer Programming
7. Proof writing
8. Communication skills: The ability to hear a description of a policy or idea or calculation or algorithm in the language of a social scientist and translate that into the language of mathematics, and vice-versa, is a valuable tool not just in the application of mathematics to the social sciences, but also to the workplace in general. This is an ideal component of a modeling or capstone course.

Undergraduate preparation for students interested in earning an advanced degree in the mathematics of social or behavioral sciences

Besides the aforementioned, a student interested in pursuing an advanced degree should also take the following courses.

1. Real analysis: An economist at the University of Michigan recently provided the unsolicited comment that real analysis is the most important mathematics class for students entering economics. Besides being applicable in many areas of theoretical economics, real analysis demands a level of rigor that is also necessary in economics.
2. Dynamical systems and Differential equations: A course in dynamical systems combined with a thorough understanding of linear ordinary differential equations is far more useful than a differential equations course focused on engineering and physical science applications.
3. Geometry and Abstract algebra: There has been an increase in the use of symmetry to understand phenomena in social and behavioral sciences.

Sample undergraduate programs

1. Mathematics and Social Sciences Honors Program at Dartmouth College

   "Mathematics and Social Sciences is an undergraduate honors major combining mathematical training with one or more of the social or life sciences. Coming from the social sciences, MSS is for students interested in Anthropology, Economics, Education, Geography, History, Political Science, or Sociology as a quantitative social science. Coming from mathematics, MSS is for students interested in statistics, data analysis, mathematics or computer sciences directed toward application in social science."

   Course offerings include: Data Analysis, Data Visualization, Game Theory, Mathematical Models in the Social Sciences, Analysis of Social Networks, Mathematical Psychology, The Meaning of Mathematical Models: Careers, Stratifications, and Mobility, Data Analysis, Models of Voting and Decision Making, and Seminars in Mathematics and Social Sciences.

   www.dartmouth.edu/~mss/

2. Applied and Computational Math Sciences at the University of Washington, with emphasis on Social and Behavioral Sciences

   "Solving complex problems in the social and behavioral sciences requires sophisticated approaches to gathering and analyzing large amounts of data. Creating mathematical models useful in prediction and analysis relies on a broad range of mathematical techniques including graph theoretical analysis, Markov chains and other stochastic processes, or differential equations. The Social and Behavioral Sciences option provides a foundation in commonly-used statistical and

computational techniques followed by flexibility in pursuing different sets of advanced courses. It is also ideal as a second major for students in social sciences who want to obtain more background in quantitative methods and mathematical techniques."

Required courses include: the Calculus sequence, Programming, ODEs, Matrix Algebra, Applied Linear Algebra and Numerical Analysis, Discrete Modeling, Probability and Statistics, Continuous Modeling, a number of electives in Physics, Biology, Computing, Analysis, and Applied Mathematics, and at least three courses in the behavioral of social science of interest.

https://www.math.washington.edu/acms/programoptions7.html

3. Applied Mathematics for the Life and Social Sciences at Arizona State University

"Students learn how to use theories and techniques such as mathematical modeling and computational methods to resolve practical real-world problems, from climate change to disease pandemics. The coursework is directed towards an understanding of mathematical theory and its relation to other fields of studies. Emphasis is placed on precision of definition, reasoning to accurate conclusions and analyzing and developing solutions to important 21st-century problems using mathematical principles."

Required courses include: two semesters of Programming, two semesters of General Biology, three semesters of Calculus, Differential Equations, Linear Algebra, Intro to Applied Math for the Life and Social Sciences, Modeling in the Life and Social Sciences, two Life Science courses, two Social Sciences courses, two applied math courses, a capstone seminar, and three or four courses in applied math, statistics, life sciences, social sciences, or mathematics education.

https://shesc.asu.edu/undergraduate/bs-applied-mathematics-life-and-social-sciences

4. Mathematical Methods in the Social Sciences at Northwestern University

"A central feature of modern social, behavioral, managerial and policy sciences is the use of mathematics and statistics, both as languages and as methods of abstraction and analysis. The faculty at Northwestern use a mathematical approach to social science in their research, and mathematics is a major component of graduate programs in the social sciences. However, most traditional undergraduate programs in the social sciences do not incorporate mathematical approaches in an organized and consistent manner. Consequently, undergraduate students in traditional programs do not receive systematic preparation in mathematical modeling.

"In the MMSS program, students receive the training needed to build and analyze mathematical models. Graduates of this program have been extraordinarily

successful as graduate students and professionals because of their rigorous preparation for examining situations with mathematical analysis."

Core Curriculum: A three-quarter sequence covering Linear Algebra and Multivariate Calculus, Foundations of Higher Mathematics, Probability and Statistics, two quarters of Econometrics, Intermediate Microeconomics, Introduction to Game Theory, Formal Models in Political Science, Advanced Game Theory, and Topics in Formal Models in Social Science.

http://www.mmss.northwestern.edu

5. Mathematics with a program specialization in Mathematical Economics at the University of Michigan

"One definition of economics is the study of the optimal allocation of scarce resources. Several mathematical techniques are fundamental to this study: constrained optimization using Lagrange multipliers, *n*-dimensional calculus, especially the Implicit Function Theorem (dependence of a solution on parameters), dynamics, probability and statistics to deal with inherent uncertainty, game theory to deal with decisions in which the actions of one agent affect the options of others, and proofs for understanding the derivation of economic principles."

Students must take courses in differential equations, discrete math or modern algebra, analysis and probability, at least three courses from a collection of applied math, financial math, or mathematical economics courses, and two additional courses.

http://www.lsa.umich.edu/math/undergrad/majorandminorprograms/infinmajorprograms/mathematicalsciences#e

6. Mathematical Economics at Wake Forest University

"The Department of Mathematics and the Department of Economics offer a joint major leading to a Bachelor of Science degree in mathematical economics. This interdisciplinary program offers the student an opportunity to apply mathematical methods to the development of economic theory, models, and quantitative analysis."

The major requires at least seven courses in economics as well as core courses in mathematics including Linear Algebra, Optimization Theory, and Dynamical Systems.

http://college.wfu.edu/math/for-undergraduate-students/undergrad-requirments

Sample graduate programs

Listed below are two graduate programs designed for students to earn advanced degrees in the mathematics of behavioral and social sciences. There are, also, voting and social choice theorists scattered throughout various mathematics graduate programs across the U.S., and it is possible to do a graduate degree in mathematics with a focus on social and behavioral science applications if a student finds the right advisor.

1. The Institute for Mathematical Behavioral Sciences at UC Irvine offers a PhD emphasizing Games, Decisions, and Dynamical Systems.

> "The Institute for Mathematical Behavioral Sciences (IMBS) in the School of Social Sciences is a specialized research center where the objective is to facilitate interaction and common research goals among scientists whose purpose is to formulate precisely and test theories of human behavior; this is to be interpreted in a wide sense as manifested by the membership which spans the following areas: anthropology, cognitive science, economics, engineering, logic and the philosophy of science, mathematics, political science, and sociology. Additional faculty come from management science and psychobiology.
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> "To describe our focus, consider the fruitful symbiotic relationship that has existed for millennia between mathematics and the physical sciences. A goal of the IMBS is to generate a similar relationship between mathematics and the behavioral and social sciences. With high-power social scientists (several are members of the National Academy) providing insights about the field and working with the mathematicians who are involved, new mathematical approaches to analyze these issues are being developed and new kinds of mathematical questions are being raised."
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> http://www.imbs.uci.edu

2. Applied Mathematics for the Life and Social Sciences at Arizona State

> "In a world beset by rapidly accelerating social and environmental dilemmas, there is urgent need for a large pool of scientists who are capable of applying advanced quantitative and qualitative approaches who also have deep insights into the challenges of societal and environmental dynamics. The applied mathematics for the life and social sciences doctoral degree is designed especially for students who want to make innovative and far-reaching scientific contributions to the global challenges of our time.
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> "Students learn a way of thinking that builds upon foundations established in mathematical epidemiology, genomics, population dynamics, bioinformatics, ecology, computational sciences, mathematical analysis and the social science fields. They experience real interdisciplinary learning that interweaves theory, applications and analytical approaches with cross-disciplinary and international collaboration. Students with strong and potentially diverse scientific and cultural

backgrounds will find a natural home in this field."

https://shesc.asu.edu/graduate/phd-program/applied-mathematics-life-and-social-sciences

Sample textbooks:

Charalambos D. Aliprantis and Subir K. Chakrabarti, *Games and Decision Making*, Oxford University Press, 2010.

Christoph Börgers, *Mathematics of Social Choice: Voting, Compensation, and Division,* Society for Industrial and Applied Mathematics, 2009.

Steven J. Brams, *The Presidential Election Game,* A. K. Peters, 2007.

Steven J. Brams, *Game Theory and Politics*, Dover Publications, 2011.

Steven J. Brams, *Game Theory and the Humanities: Bridging Two Worlds,* MIT Press, 2012.

Steven J. Brams and Peter C. Fishburn, *Approval Voting,* Springer, 2007.

Steven J. Brams and Alan D. Taylor, *Fair Division: From Cake-Cutting to Dispute Resolution,* Cambridge University Press, 1996.

Alpha Chiang and Kevin Wainwright, *Fundamental Methods of Mathematical Economics*, McGraw-Hill, 2004.

Drew Fudenberg and Jean Tirole, *Game Theory,* MIT Press, 1991.

Jonathan K. Hodge and Richard E. Klima, *The Mathematics of Voting and Elections: A Hands-On Approach,* American Mathematics Society, 2005.

Earl Hunt, *The Mathematics of Behavior,* Cambridge University Press, 2006.

Roger B. Myerson, *Game Theory: Analysis of Conflict,* Harvard University Press, 1997.

Donald G. Saari, *Chaotic Elections! A Mathematician Looks at Voting*, AMS, 2001.

Donald G. Saari, *Decisions and Elections: Explaining the Unexpected*, Cambridge University Press, 2001.

Donald G. Saari, *Disposing Dictators, Demystifying Voting Paradoxes,* Cambridge University Press, 2007.

Philip D. Straffin, *Game Theory and Strategy*, The Mathematical Association of America, 1993.

Alan D. Taylor, *Social Choice and the Mathematics of Manipulation*, The Mathematical Association of America, 2005.

Alan D. Taylor and Allison M. Pacelli, *Mathematics and Politics: Strategy, Voting, Power, and Proof,* Springer, 2009.

Sample journals:

Mathematical Social Sciences, Elsevier (since 1980)

"Topics of particular interest include the fundamental aspects of choice, information, and preferences (decision science) and of interaction (game theory and economic theory), the measurement of utility, welfare and inequality, the formal theories of justice and implementation, voting rules, cooperative games, fair division, cost allocation, bargaining, matching, social networks, and evolutionary and other dynamics models."

Social Choice and Welfare, Springer (since 1984)

"Social Choice and Welfare explores all aspects, both normative and positive, of welfare economics, collective choice, and strategic interaction. Topics include but are not limited to: preference aggregation, welfare criteria, fairness, justice and equity, rights, inequality and poverty measurement, voting and elections, political games, coalition formation, public goods, mechanism design, networks, matching, optimal taxation, cost-benefit analysis, computational social choice, judgement aggregation, market design, behavioral welfare economics, subjective well-being studies and experimental investigations related to social choice and voting. As such, the journal is interdisciplinary and cuts across the boundaries of economics, political science, philosophy, and mathematics."

Journal of Mathematical Psychology, Elsevier (since 1964)

"Areas of special interest include, but are not limited to, fundamental measurement and psychological process models, such as those based upon neural network or information processing concepts. A partial listing of substantive areas covered include sensation and perception, psychophysics, learning and memory, problem solving, judgment and decision-making, and motivation."

Theory and Decision, Springer (since 1971)

"Theory and Decision is devoted to all aspects of decision-making, exploring research in psychology, management science, economics, the theory of games, statistics, operations research, artificial intelligence, cognitive science, and analytical philosophy. Moreover, it addresses cross-fertilization among these disciplines."

Electoral Studies, Elsevier (since 1982)

"Electoral Studies is an international journal covering all aspects of voting, the central act in the democratic process. Political scientists, economists, sociologists, game theorists, geographers, contemporary historians and lawyers have common, and overlapping, interests in what causes voters to act as they do, and the consequences. *Electoral Studies* provides a forum for these diverse approaches."

Journal of Theoretical Politics, Sage (since 1989)

*"*The *Journal of Theoretical Politics* is an international peer reviewed journal one of whose principal aims is to foster the development of theory in the study of political processes. JTP provides a forum for the publication of original papers seeking to make genuinely theoretical contributions to the study of politics."

Public Choice, Springer (since 1966)

"Public Choice studies the intersection between economics and political science. The journal plays a central role in fostering exchange between economists and political scientists, enabling both communities to explain and learn from each other's perspectives. This journal's roots are in the application of economic methods to problems normally dealt with by political scientists. While it retains strong traces of

economic methodology, currently it also addresses newly developed, effective techniques that are not within the domain of economists."

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