Financial Mathematics

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Introduction

Financial Mathematics developed in the mid-1980s as research mathematicians became interested in problems, largely involving stochastic control, that had until then been studied primarily by economists. The subject grew slowly at first and then more rapidly from the mid-1990s through to today as mathematicians with backgrounds first in probability and control, then partial differential equations and numerical analysis, got into it and discovered new issues and challenges. A society of mostly mathematicians and some economists, the Bachelier Finance Society, began in 1997 and holds biannual world congresses. The Society for Industrial and Applied Mathematics (SIAM) started an Activity Group in Financial Mathematics & Engineering in 2002; it now has about 800 members. The 4th SIAM conference in this area was held jointly with its annual meeting in Minneapolis in 2013, and attracted over 300 participants to the Financial Mathematics meeting. In 2009 the SIAM *Journal on Financial Mathematics* was launched and it has been very successful gauged by numbers of submissions.

Student interest grew enormously over the same period, fueled partly by the growing financial services sector of modern economies. This growth created a demand first for quantitatively trained PhDs (typically physicists); it then fostered the creation of a large number of Master's programs around the world, especially in Europe and in the U.S. At a number of institutions undergraduate programs have developed and become quite popular, either as majors or tracks within a mathematics major, or as joint degrees with Business or Economics. These programs are the main topic of this report.

Financial Mathematics, now a quarter-century in existence (and encompassing other common descriptors such as Mathematical Finance or Computational Finance) has become a vibrant research area of applied mathematics. It continues to attract new young researchers, and mathematics departments worldwide are creating positions in the field, both to interact with their applied mathematics groups and to meet teaching demand. Undergraduates want to learn the mathematics of uncertainty, for which they see both immediate applications and potential demand from employers. Indeed, the aftermath of the Financial Crisis, while rightly decreasing the incentive for and scope of excessive risk taking, has increased the relative demand for quantitative training at all levels. After each disaster, the industry demands more mathematics, not less.

Cognitive goals

Financial Mathematics is an ideal area for providing a broad view of the mathematical sciences. Building on a foundation of analysis and discrete mathematics, financial mathematics draws on discrete and continuous probability and random processes, optimization, dynamical analysis, ODE and PDE, and numerical analysis. Links to the allied fields of Statistics, Finance, Computing, and Economics are central to this highly interdisciplinary area. Because the field is driven by analysis of financial data, effective use of computer programming is essential, especially for careers in the financial industry. These needs should be addressed by careful course selection in these various fields, along with specialized courses in financial mathematics that focus on current mathematical and numerical approaches to problems from finance and economics.

While students in a Financial Mathematics program will typically be among the most interdisciplinary of mathematics majors, pure mathematical understanding of probability and analysis is required. The curriculum should therefore support broad achievement: in pure mathematical thinking, computation, and applications.

The typical target group will be students interested in careers related to quantitative finance in industry and government, either before or after further graduate study, as well as those aiming for academic or industry research careers.

Content goals

Students who pursue financial mathematics should achieve several key mathematical goals and outcomes.

1. Post-calculus mathematics majors (and minors) should be exposed to the mathematics of randomness with a rigorous but applied probability course.

2. Students should learn about discrete models of asset pricing, with a later less rigorous treatment of continuous-time models based on Brownian motion.

3. Students need modern computational skills motivated by applications in finance and economics. Techniques include tree methods, finite differences for PDEs and Monte Carlo simulation. The latter is used extensively in the industry and could be developed as a class in itself with financial and other applications.

4. A significant number of mathematics majors with interest in finance and economics should be encouraged and prepared for graduate school (PhD) in this area. This would mean, in addition to the usual classes, strong training in real analysis. Increasing the

number of U.S. citizen graduate students in Mathematics and related fields is a desirable outcome that is supported by a financial mathematics track.

5. Students should be afforded opportunities to pursue data-driven research projects with financial applications, during summers or as part of an independent class. With the availability of high-frequency data, this provides good exposure to the challenges and techniques of Big Data analysis, which skills are currently in great demand in other modern industries.

Prerequisites for graduate work (at MS and PhD levels)

Because of the high level of mathematical background required to address the main questions in financial mathematics, the core topics are usually treated most fully in graduate school. The role of the undergraduate program should be to (1) provide adequate mathematical background; (2) expose students to the allied fields of statistics, computing, finance, and economics; and, optionally, (3) expose the student to some core ideas and problems of the field. We describe these goals further in what follows.

Mathematical background. In the lower division students need multivariable calculus, linear algebra, and ordinary differential equations. Students should also take one or two courses in probability and statistics, and one or two semesters of proof-based advanced calculus or real analysis. Additional proof-based courses in topology, discrete mathematics, or other topics would be useful to develop mathematical maturity. Courses in PDE and numerical analysis would also be helpful.

Exposure to allied fields. The most important skill for industry and government employment is computer programming. Graduate study requires a working knowledge of a programming language sufficient for scientific computing, such as C++, Fortran, Java, or Python. Sometimes MATLAB or a similar package is adequate for graduate courses, though not reliably for industry employment. C++ still seems to be the preferred language for employment of quants, though Python is gaining ground and some firms use alternatives like C#.

Students should take an introduction to statistics if that is not already a part of the probability courses mentioned above. An introduction to Corporate Finance or Investments from the Finance department is sometimes needed. One or more introductory courses in economics are also recommended.

Topic-oriented courses. If available, a mathematical course introducing students to the problems of quantitative finance would be helpful to undergraduate majors.

Program-specific courses needed

There are a few courses departments should consider offering that may not already be a part of the core mathematics program.

1. An introduction to Financial Mathematics, with probability, and possibly differential equations, as prerequisite. This class would discuss discrete pricing models, arbitrage, derivatives, and a gentle treatment of continuous-time models and Brownian motion, and an optional treatment of the Black-Scholes equation.

2. A computational finance course, using languages such as Python, C++, Java, Fortran, or packages like MATLAB to address basic topics in numerical analysis. The course can cover fundamental techniques for solving systems of linear equations, Newton-type methods, numerical approximations to derivatives and integrals and numerical approximations to solutions of ODEs, while including computing. Students should be able to write simple programs and understand how computers store numbers, introduce errors, and present other pitfalls. These methods can be applied to pricing problems using tree methods, finite differences for PDEs, and/or Monte Carlo simulation.

3. A mathematical time series course, to give students a better understanding of how stochastic processes are modeled. This is also a core topic in mathematical economics.

4. A course in mathematical economics, either within the Economics department, or with the collaboration of Economics faculty, helps students understand the language and perspective of economics, and introduces important problems and methods in that field.

Sample undergraduate financial mathematics programs

Many existing undergraduate programs that involve mathematics and finance or mathematics and economics. A list of twenty schools with links describing their minor, concentration, track, major or double major is in the appendix. All of these programs have courses and requirements that adhere to the guidelines mentioned in the preceding two sections. All of the programs require students to take courses both in mathematics and in finance and/or economics. Some of the programs have interdisciplinary courses in mathematics and finance. These programs generally require the students to take enough mathematics, statistics, probability, finance and economics to provide value not only to the financial sector, but, also to analytical jobs in business, industry and government, or, optionally, to prepare for graduate school. In more detail:

Mathematics courses. All of the universities in the appendix require the standard calculus sequence, linear algebra, and ODEs for the minor, the concentration, the track, the major, and the double major. Several of the minors, concentrations and tracks require a numerical computing course that emphasizes matrix algebra, solving systems of equations and numerical calculus. Some of the tracks and concentrations require probability and statistics. The majors and double majors require the numerical computer course, probability and statistics. Some of the probability courses cover Monte Carlo methods and Brownian motion. The universities offering majors and double majors usually require a course in analysis or advanced calculus, an upper level (linear) algebra course and sometimes a course in partial differential equations. Some of the programs also offer courses in time series analysis and courses in discrete, continuous, and stochastic dynamics.

Finance courses. A basic finance course covering the time value of money, fixed income, pricing bonds and equity is standard in these programs. Courses in micro- and macro-economics are required or recommended in most of the programs. The major and double major programs often have a course in options modeling using binomial trees. Sometimes the major and double major programs introduce stochastic calculus (including Ito's lemma) and the Black-Scholes model.

Computing courses. Many of the cited programs require a computing course in C++ or Python.

Capstone experiences. Some of the listed programs require a capstone course in which students do a data-driven project or a case study in economics or finance and write a report that includes a mathematical model. Many of these capstone courses are cross-listed with a Finance or Economics department. Some of the programs team-teach these courses, which exposes students to views and ideas of professors from more than one discipline.

A mathematics department contemplating an undergraduate mathematical finance program for the first time is advised to start off with a minor, a concentration, or a track. After gaining experience, building relations with Finance and attracting students, a major or double major is then an easier step to take.

Selected textbooks

Most of these are textbooks used by various audiences at the advanced undergraduate and beginning graduate levels, and are listed here as a set of suggestions to use as starting points for building a course. In some cases there may be more recent editions than those indicated.

1. Goodman & Stampfli, *The Mathematics of Finance* (2001, AMS)
2. Tuckman, *Fixed Income Securities* (2011, Wiley)
3. Fabozzi, *Bond Markets, Analysis and Strategies* (2012, Prentice-Hall)
4. Fabozzi, *Interest Rates, Term Structure, and Valuation Modeling* (2002, Wiley)
5. Hull, *Options, Futures, and Other Derivatives* (2011, Prentice Hall)
6. Baxter and Rennie, *Financial Calculus* (1996, Cambridge)
7. Wilmott, Howison, Dewynne, *The Mathematics of Financial Derivatives* (1995, Cambridge)
8. Shreve, *Stochastic Calculus for Finance I* (2004, Springer)

Appendix: Following are selected institutions with existing undergraduate programs or options in Mathematical Finance, Financial Mathematics, or Mathematical Economics. Much more information is available online from these institutions' websites.

- University of Richmond
- Temple University
- University of California - San Diego
- Carnegie Mellon University
- James Madison University
- Ball State University
- Fordham University
- University of California - Santa Cruz (joint degree in Math and Econ)
- London School of Economics
- New Jersey Institute of Technology
- Simmons College
- University of Michigan
- Wilfrid Laurier University Canada
- University of Houston
- North Carolina State University
- University of Pittsburgh
- Princeton University
- University of Victoria
- University of Florida
- University of Mount Union